Symmetric Relations

D. Terence Langendoen
University of Arizona

1. Full symmetry

Logical symmetry is standardly defined for two-place relations only, as in (1).

(1) $S$ is symmetric iff $S(x, y)$ is equivalent to $S(y, x)$.

While (1) provides an adequate account of the symmetry of the relation be similar that appears in (2a), it does not do so for the relations that appear in (2b)-(2d).

(2) a. Bush and Robertson are similar.
   b. Bush, Robertson and Haig agreed.
   c. The Republican candidates for President separated.
   d. The natural numbers are distinct.

Example (2b) expresses a proposition which relates the three persons Bush, Robertson and Haig. Similarly, (2c) relates an unspecified plural number of persons; and (2d) relates a countable infinity of abstract objects. The question therefore arises as to how the notion of symmetry given in (1) should be generalized to cover relations with two or more places. A natural generalization, which we call full symmetry, requires equivalence among all orderings of the arguments of an $n$-place relation $S_n(n > 1)$, as in (3)

(3) $S_n(n > 1)$ is fully symmetric iff $S_n(x_1, \ldots, x_n)$ is equivalent to $S_n(p(x_1, \ldots, x_n))$, where $p(x_1, \ldots, x_n)$ is any permutation of $x_1, \ldots, x_n$.

By quantifying over $n$ in the definition of full symmetry in (3), as in (4a), we approach a definition of the notion of a family of fully symmetric relations.

(4a) $S_n$ is a family of fully symmetric relations iff, for all $n > 1$, $S_n(x_1, \ldots, x_n)$ is equivalent to $S_n(p(x_1, \ldots, x_n))$, where $p(x_1, \ldots, x_n)$ is any permutation of $x_1, \ldots, x_n$. 
However, (4a) by itself is not sufficient, inasmuch as it does not guarantee that the various relations in the family have anything in common besides their symmetry. What is required is in addition a stipulation like that in (4b) in which each \( n \)-place relation in the family implies every \( m \)-place relation in the family for \( 1 < m < n \).

(4b) Furthermore, for all \( m, n \) such that \( 1 < m < n \) and such that the arguments in \( S_m \) are a subcollection of the arguments in \( S_n \), \( S_n \) implies \( S_m \).

A family of relations is fully symmetric if it satisfies both clauses of (4). A large number of the English words and phrases that have been analyzed as symmetric relations in the literature and other places (Gleitman 1965, Lakoff and Peters 1969, Quang c1970, Langendoen 1978) express families of fully symmetric relations as defined in (4). We illustrate using the relation be similar.

(5) a. Bush and Robertson are similar.
   b. Robertson and Bush are similar.
   c. Bush and Haig are similar.
   d. Haig and Bush are similar.
   e. Robertson and Haig are similar.
   f. Haig and Robertson are similar.
   g. Bush, Robertson and Haig are similar.
   h. Bush, Haig and Robertson are similar.
   i. Robertson, Bush and Haig are similar.
   j. Robertson, Haig and Bush are similar.
   k. Haig, Bush and Robertson are similar.
   l. Haig, Robertson and Bush are similar.

Clearly, the propositions expressed in (5a) are equivalent, and similarly those in (5b), (5c) and (5d). Further, as the cardinality of the subject expression increases, the corresponding equivalences continue to hold. Thus the requirement in (4a) is met. Clearly also, each of the propositions expressed in (5d) imply all of those in (5a-c). Again, as the cardinality of the subject expression increases, the corresponding implications continue to hold. Thus the requirement in (4b) is also met.

If attack is substituted for be similar in (5) the equivalence and implication relations just described also appear to be maintained. However, we do not consider attack to be a fully symmetric relation, holding of the individual which comprise its subject as separate arguments. Rather, in these occurrences, attack is a one-place predicate which holds of its subject as a whole. The most obvious distinction between symmetric relations and one-place predicates is that the latter can hold of individuals, as in (6a), whereas the former cannot, as shown by the ungrammaticality of (6b).

   b. *Bush is similar.

Moreover, nonsymmetric relations like attack are fully additive over the collections that comprise its arguments (Eilenberg 1974:2). In particular, all of the sentences in (7) are logically equivalent.

(7) a. Bush attacks, Robertson attacks and Haig attacks.
   b. Bush and Robertson attack, Bush and Haig attack and Robertson and Haig attack.
   c. Bush, Robertson and Haig attack.

On the other hand, if be similar is substituted for attack in (7), (7a) is ungrammatical, and (7b) does not imply (7c).

2. Placedness

The 'variable polyadicity' (Davidson 1967) of fully symmetric relations differs significantly from that of nonsymmetric predicates and relations such as attack in (8).

   b. Bush attacks Robertson.
   c. Bush attacks Robertson with a hymnbook.
   d. Bush attacks Robertson in South Carolina.
   e. Bush attacks Robertson in South Carolina with a hymnbook.

In the examples in (8), the various entities related by attack occupy distinct grammatical positions, and there may be no more than finitely many of these positions associated with any particular occurrence of the relation. We analyze the placedness of nonsymmetric relations like attack by means of a method originally devised by Tarski (1933).

(9) Let \( q = <x_1, x_2, \ldots> \) be an infinite sequence of entities, and let \( Q \) be the collection of all such sequences. Let \( R \) be a mapping of elements of \( Q \) onto a collection of sentences \( S \). Define the support of \( R \) as the set of places in the sequence which have an effect on the mapping; that is, when different entities are substituted in those places, different sentences are mapped onto. Then the cardinality \( c \) of the support of \( R \) is the placedness of \( R \). Like Tarski, we assume \( c \) is finite.

Let us call each \( q \) in (9) a placedness sequence. We assume that finitely many positions in each placedness sequence are associated with particular argument types, which can be thought of as grammatical relations.
such as subject, direct object, etc.; as role or thematic types, such as agent, theme, etc.; or as a mixture of the two. We adopt the second method of analysis, and associate subject with position 1, direct object with position 2, indirect object with position 3, source with position 4, goal with position 5, instrument with position 6, etc.; locative with position 20, temporal with position 21, etc. The break in the numbering of sequence positions is intended to correspond to the division of places into argument and adjunct positions; the listing is suggestive and not intended as definitive. For purposes of lexical analysis, it is desirable to modify Tarski’s method by breaking each placedness sequence into two subsequences: an argument subsequence $q_1$, containing elements for which lexical items subcategorize; and an adjunct subsequence $q_2$, containing elements for which they do not, as in HPSG (Pollard and Sag 1987). The relation attack and other nonsymmetric relations would apply to $q_1$ and the result, instead of being a sentence, would be another relation, which would apply to $q_2$, yielding a sentence.11

The entities which occupy the various positions in placedness sequences may be individuals, collections of individuals or variables bound by quantifiers. A variable bound by an indefinite quantifier with narrow scope is associated with a null expression (symbolized $e$) in specific positions in English and other languages (Fodor and Sag 1982).

To determine the support of the relation expressed by attack in English, consider the sequences in (10).

\begin{itemize}
  \item \textbf{a.} i. \textless c, e, c, ... \textgreater 
  \item ii. \textless Bush, c, e, ... \textgreater 
  \item iii. \textless Robertson, c, e, ... \textgreater 

  \item \textbf{b.} i. \textless c, Bush, e, ... \textgreater 
  \item ii. \textless Bush, Bush, e, ... \textgreater 
  \item iii. \textless Robertson, Bush, c, ... \textgreater 
  \item iv. \textless c, Robertson, e, ... \textgreater 
  \item v. \textless Bush, Robertson, c, ... \textgreater 
  \item vi. \textless Robertson, Robertson, e, ... \textgreater 

  \item \textbf{c.} i. \textless e, c, Bush, ... \textgreater 
  \item ii. \textless Bush, c, Bush, ... \textgreater 
  \item iii. \textless Robertson, e, Bush, ... \textgreater 
  \item iv. \textless e, Bush, Bush, ... \textgreater 
  \item v. \textless Bush, Bush, Bush, ... \textgreater 
  \item vi. \textless Robertson, Bush, Bush, ... \textgreater 

\end{itemize}

The ungrammaticality of (11ai), (11bi), and (11biv) is a consequence of the restriction that $e$ (as described above) cannot occur in subject position in indicative English sentences, and need not concern us further. From the mapping of the sequences in (10) onto the sentences in (11), we see that positions 1 and 2 (subject and direct object) are part of the support of the relation attack, but not position 3 (indirect object). Varying the entities that appear in positions 1 and 2 results in different sentences being mapped onto, but varying the entities that appear in position 3 has no effect on the mapping. A more detailed analysis would reveal that positions 4 and 5 (source and goal) are also not part of the support of the relation, but that positions 6, 20 and 21 (instrument, locative and temporal) are.12

The occurrence of plural, including conjoined, expressions in the various positions corresponding to particular argument types has no effect on the number of places associated with nonsymmetric relations. For example, in the sentences in (12), the number of places associated with attack is the same as in the corresponding sentences in (8).

\begin{itemize}
  \item \textbf{a.} Bush and Robertson attack.
  \item \textbf{b.} Bush and Robertson attack Haig and Dole.
  \item \textbf{c.} Bush and Robertson attack Haig and Dole with hymnbooks.

\end{itemize}
noting single entities cannot occur in those positions in those sentences.\textsuperscript{14} Second, note that, for example, (13) is not implied by the conjunction in (16).

(16) Quayle separated Bush and Robertson; Quayle separated Bush and Haig; and Quayle separated Robertson and Haig.

In (13), then, each of the entities denoted by Bush, Robertson, and Haig are related to the others and to the entity denoted by Quayle. Similar observations can be made concerning (14) and (15).

Although multisets can occur in two distinct positions in placedness sequences, at most one multiset can appear as an element of any particular sequence. That is, there are no lexical items which subcategorize for two or more multisets, nor can two or more multisets be associated with any lexically complex relation. For example, there is no relation like \textit{glarf} in (17) which subcategorizes for a multiset in both subject and direct object or goal positions.

(17) Bush, Robertson and Haig glarf (into) Dole and DuPont.

Sentences like (18) are not counterexamples to this principle.

(18) Bush, Robertson and Haig are similar to Dole and DuPont.

In (18), the subject expression Bush, Robertson and Haig and the goal expression Dole and DuPont as wholes are arguments of be similar; that is, be similar in (18) is a two-place relation, and Bush, Robertson, Haig, Dole and DuPont do not separately constitute arguments of that relation. In (18), the conjoined expressions Bush, Robertson and Haig and Dole and DuPont behave just as plural expressions do in nonsymmetric relations as exemplified in (12). Thus, not only does the argument sequence for be similar in (18) not contain two multisets of arguments, it does not even contain one.\textsuperscript{15}

The interpretation of sentences like (18) shows that there are additional restrictions on the conditions under which a single multiset of arguments can appear in a placedness sequence. The first restriction is that if a lexical item subcategorizes for a subject or direct-object multiset, then it does not subcategorize for a source or goal argument. The nonoccurrence of subject multisets with source or goal arguments is illustrated by (18); the nonoccurrence of direct-object multisets with source or goal arguments is illustrated by (19).

(19) Quayle separated Bush, Robertson and Haig from Dole and DuPont.

In (19), the compound entity denoted by Bush, Robertson and Haig occurs directly as an argument of the relation separate; i.e., it behaves like the sub-

d. Bush and Robertson attack Haig and Dole in eleven states.

e. Bush and Robertson attack Haig and Dole in eleven states with hymnbooks.

In (12d), for example, the entity denoted by Bush and Robertson occupies the subject position in the placedness sequence, that denoted by Haig and Dole the direct object position, and that denoted by eleven states the locative position.

Returning now to the analysis of fully symmetric relations, we note that the expressions which denote the arguments of the relations in (2) and (5) all occupy the same grammatical position, that of subject. This observation suggests the following modification of Tarski's method of analyzing placedness for families of fully symmetric relations: the subject position in the placedness sequence may itself be occupied by a multiset of at least two entities, and each entity in the multiset counts as an argument of the relation. This modification has a number of advantages. First, using multisets in this way accounts directly for the fact that permuting the elements that appear in subject position has no semantic consequences. Second, a multiset, like a set, can have any number, finite or infinite, of members; hence the possibility of a relation having infinitely many places, as in (2d), is accounted for. Third, since the multiset as a whole occupies a position in the placedness sequence, it, not the entities that make it up, is correctly analyzed as having the grammatical function of that position (subject). Thus, for example, in the case of (5d), the entities denoted by Bush, Robertson and Haig are separate arguments of the relation be similar, but the corresponding expressions are not themselves subjects of that sentence. Only the entire expression Bush, Robertson and Haig is.

A multiset of arguments can also appear in direct object position as in (13); in goal position, possibly together with a direct object, as in (14); or in subject position, together with a direct object, as in (15).

(13) Quayle separated Bush, Robertson and Haig.

(14) a. The compound separated into its component elements.

b. The chemist separated the compound into its component elements.

(15) Bush, Robertson and Haig shared a hymnbook.

The various relations expressed by separate in (13) and (14), and by share in (15) are not fully symmetric, since not all the results of permuting the entities denoted by their arguments (Quayle, Bush, Robertson and Haig in (13); the compound and the individual elements comprising its component elements in (14a); and the chemist, the compound and the individual elements comprising its component elements in (14b); and Bush, Robertson, Haig and a hymnbook in (15)) are equivalent. However, the entities occurring as conjuncts in direct object position in (13), and those making up the plural expression in goal position in (14) are separate arguments of the relations occurring in those examples. To see this, first note that expressions de-
ject in (18), not like the direct object in (13). The second restriction is that a subject multiset does not occur together with an comitative argument, as in (20).

(20) Bush, Robertson and Haig shared a hymnbook with Dole and DuPont.

In (20), just as in (18), the individuals denoted by *Bush, Robertson and Haig* do not occur as separate arguments of *share*; only the compound entity denoted by *Bush, Robertson and Haig* does.

At the moment, we have no explanation for the fact that for nearly every relation that selects a multiset in a given position, there is a morphologically identical counterpart that selects an ordinary argument in that position, together with an ordinary source, goal or comitative argument not found in the support of the original relation.

3. Derived symmetry

Syntactically complex expressions may also select multisets of arguments in particular positions, as (21) illustrates.

(21) Bush, Robertson and Haig have several friends in common.

In (21), a multiset may be analyzed as occurring in subject position of the complex relation have ... in common. In contrast, the subject of the lexically simple relation have in (22) is occupied by an expression referring to a single, compound entity.

(22) Bush, Robertson and Haig have several friends.

Alternatively, the phrase in common could be analyzed as an operator on the phrase have several friends, with the resulting complex relation have several friends in common in turn selecting a subject multiset. The latter analysis, however, requires that the subject not occupy a position in the placedness sequence for lexical relations, but rather that it be the single position of a syntactically complex relation obtained from the head lexical relation by filling or binding each of its nonsubject places (cf. note 11). We reject this alternative analysis, however, because the operator that helps select a multiset can also occur as part of an argument or adjunct expression anywhere in the placedness sequence, including subject position. One such operator is the adjective same, as in the examples in (23).17

(23) a. Bush, Robertson and Haig attacked the same person.
   b. Bush, Robertson and Haig attacked Dole in the same state.
   c. Bush attacked Robertson, Haig and Dole with the same hymnbook.

We conclude from these observations that the selection of multisets may be made by any lexical relation together with an operator, such as in common and same, occurring either as an adverbial modifier of that relation, or as an element of an argument or adjunct of that relation.18 In conformity with the principle that simple lexical relations can select at most one multiset, complex ones are also limited to selecting at most one multiset. Consider the examples in (24).

(24) a. Bush; Robertson and Haig attacked Dole and DuPont in the same state.
   b. Bush, Robertson and Haig attacked Dole and DuPont in the same state on the same day.

These examples can be interpreted with a multiset of arguments in either subject or direct object position (i.e., they are ambiguous with respect to where the multiset occurs), but not in both. No matter how many times an operator helps select a multiset occurs, only one multiset can be introduced. In this regard, the operators that help select multisets are comparable to the asymmetric comparative operator more, which helps introduce exactly one new source argument, no matter many times it occurs.19

(25) a. Bush attacked Robertson in more states more often than Haig (*than Dole).
   b. i. More opponents attacked Bush more often with a hymnbook than Robertson.
       ii. More opponents attacked Bush more often with a hymnbook than with a prayerbook.
       iii. *More opponents attacked Bush more often with a hymnbook than Robertson than with a prayerbook.

Note that (25a), when grammatical, is ambiguous, as than Haig can be analyzed as a source either in relation to the subject (Bush) or the direct object (Robertson).20

NOTES

1. More precisely, we should say that example (2b) expresses a proposition containing a relation among three individuals only on one of its interpretations. On another, it expresses a proposition containing a one-place predicate, in which the three individuals together are said to agree with an unnamed individual or institution. Similar
ambiguities arise in nearly all of the examples used in this paper to illustrate symmetric relations.

2. Other extensions of the standard definition of symmetry in (1) are also possible, a few of which, such as reciprocity (see, for example, Fiengo and Lasnik 1973 and Langendoen 1978) are linguistically significant. However, the exploration of these extensions is beyond the scope of this paper.

3. We omit mention of the individual arguments of the relations $S_a$ and $S_m$, since, in virtue of (3), they may occur in any order in each of them.

4. Example (6b) is grammatical on a different, nonsymmetric, interpretation, in which the subject is being compared to an unnamed individual; cf. note 1.

5. Essentially this point was made by Leonard and Goodman (1940); see Langendoen (1978:189-91) for discussion. My notion of strong reciprocity for subsets in that article was intended to express what is here more accurately expressed as the notion of full symmetry. Note, however, that the counterparts to (7b) and (7c), with be identical substituted for attack, are logically equivalent.

6. For ease of exposition, we henceforth refer to one-place predicates, such as attack in (8a), as one-place relations.

7. Fully symmetric relations can also be extended in this way, compare (2b) with (i).

(i) a. Bush, Robertson and Haig agreed in South Carolina.
   b. Bush, Robertson and Haig agreed in South Carolina last Thursday.

Strictly speaking, the examples in (i) do not express symmetric relations, since South Carolina and last Thursday cannot be permuted with Bush, Robertson and Haig. However, the complex relations agree in South Carolina and agree in South Carolina last Thursday can be considered to be fully symmetric relations.

8. I thank Arnold Koslow for bringing Tarski’s paper to my attention.

9. We assume that the sequence as a whole is infinite even though only finitely many positions in it are used to provide support for any particular relation. In this way, we can provide for more position types as needed without running out of positions for them to occupy.

10. However, some position types would then have to belong to argument subsequences with respect to certain relations and to adjunct subsequences with respect to others. For example, the locative position type is an adjunct position with respect to attack, but an argument position type with respect to stay.
Both operators in common and same also permit subcategorization for an additional place of the source, goal or comitative type, as in the following examples.

(i) Bush has several friends in common with Robertson and Haig.
(ii) Bush attacked the same person as Robertson and Haig.

In such examples, no multiset is selected for, just as when a lexical relation subcategorizes for a source, goal or comitative argument in addition to a subject or direct object, as in (18) and (20). Again, we have no explanation for this fact.

However, each operator can introduce a separate source constituent if those constituents are not nested, and if one occurrence of the operator modifies a nonsubject argument and the other modifies an adjunct, as in (6).

(i) Bush attacked more opponents than Robertson more often with a hymnbook than with a prayerbook.

I have no satisfactory explanation for this state of affairs.

Similarly, symmetric operators can help introduce at most one source, goal or accompaniment position, no matter how many times they occur, as the examples in (i) and (ii) illustrate.

(i) Bush attacked Robertson in the same state on the same day as Haig ("as Dole.
(ii) a. Bush attacked the same candidate in South Carolina on the same day as Robertson.
   b. Bush attacked the same candidate in South Carolina on the same day as in Georgia.
   c. *Bush attacked the same candidate in South Carolina on the same day as Robertson as in Georgia.

Example (i), when grammatical, is ambiguous, in the same way that (25a) is.

REFERENCES

