B. DUNHAM, R. FRIDSHAL, and G. L. SWARD. *A non-heuristic program for proving elementary logical theorems.* English, with French, German, Russian, and Spanish summaries. ***Information processing, Proceedings of the International Conference on Information Processing, Unesco, Paris 15–20 June 1959,*** Unesco, Paris, R. Oldenbourg, Munich, and Butterworths, London, 1960, pp. 282–285.

B. DUNHAM, R. FRIDSHAL, and J. H. NORTH. *Exploratory mathematics by machine.* ***Recent developments in information and decision processes,*** edited by Robert E. Machol and Paul Gray, The Macmillan Company, New York 1962, pp. 149–160.

B. DUNHAM and J. H. NORTH. *Theorem testing by computer.* ***Proceedings of the Symposium on Mathematical Theory of Automata, New York, N.Y., April 24, 25, 26, 1962,*** Microwave Research Symposia series vol. 12, Polytechnic Press of the Polytechnic Institute of Brooklyn, Brooklyn, N.Y., 1963, pp. 173–177.

These three papers are directed to the problem of efficient methods for testing, by computer, the validity of *loose, expanding* truth-functional expressions, such as are obtained in the Herbrand expansions of formulas in the first-order predicate calculus.

The Elimination Theorem in *Theorem testing by computer* is the only interesting formal result. Definition: In an expression in disjunctive normal form, an occurrence of a literal l in a clause $l \wedge A_i$ and an occurrence of its $\neg l$ in a clause $\neg l \wedge A_j$ interact if $A_i \wedge A_j$ is not a contradiction. Theorem: Every valid formula in disjunctive normal form contains a valid closed subset of clauses in which each occurrence of a literal interacts with at least one occurrence of its negation. In testing for validity, the theorem is applied by dropping from consideration all clauses containing non-interacting occurrences of literals, and thus reducing the size of the problem. The authors remark that the theorem could also serve as the basis for look-ahead in generating only relevant terms of the Herbrand expansion, but do not explore this idea further.

The method of progressive variable elimination and simplication given in *A non-heuristic program for proving elementary logical theorems* is a variant of standard methods, interesting primarily because it was actually programmed. The colorfully-described algorithm of *Exploratory mathematics by machine* tests for validity in such a way that when additional clauses are added to the formula by expansion, the process can use, without recomputation, much of the information already obtained. It requires empirical justification, but the paper disappointingly reports that "programming is not yet completed."

It is perhaps unfortunate that these papers are reviewed so long after they were written. Recent developments in theorem-proving by computer have not tended to support the underlying motivation for the work. Dunham and North argue that "in order to test whether nontrivial logical formulae are theorems, we shall probably need computer programs which either manage very large truth-functional expressions or else simplify by an efficient look-ahead the Herbrand expansion." These papers consider only the first half of this problem. Yet today the successes obtained on the look-ahead part and on alternative methods of theorem-proving have greatly reduced the importance of the problem treated. Most of the current work on proof procedures follows Prawitz XXXI 126(2) in making only those substitutions of the Herbrand universe which contribute directly to the solution (see also XXXI 468).

The papers contain some discussions of computer processing which is not relevant to this JOURNAL. JOYCE FRIEDMAN

JOSEPH ULLIAN. *Failure of a conjecture about context free languages.* ***Information and control,*** vol. 9 (1966), pp. 61–65.

The author proves that for the non-regular set $L = \{a^i b^j d^k \mid i \neq j \vee j \neq k\}$ that $N(L) = \{xcy \mid x, y \in L \wedge x \neq y\}$ is context free, thereby demonstrating the failure of a conjecture of Haines (***Generation and recognition of formal languages,*** Ph.D. dissertation, M.I.T., 1965) that $N(L)$ is never context free for non-regular L. The conjecture was originally entertained because Haines was able to prove the converse, that for regular L, $N(L)$ is context free.

The author's demonstration proceeds by considering $N(L)$ as the union of eight subsets M_i $(1 \leqq i \leqq 8)$ defined by the various inequalities holding among the exponents of the letters making up words of $N(L)$. Each of the subsets is shown to be context free by demonstrating that the exponents in sentences of M_i form a finite union of linear sets each with a stratified set of periods.

This paper clearly demonstrates the utility of the application of the theory of linear sets to the problem of determining whether or not particular languages are or are not context free.

D. TERENCE LANGENDOEN

HASKELL B. CURRY. *Combinatory logic.* ***Summaries of talks presented at the Summer Institute for Symbolic Logic, Cornell University, 1957,*** 2nd edn., Communications Research Division, Institute for Defense Analyses, Princeton, N.J., 1960, pp. 90–99.

A very condensed abstract by Rosser of two lectures by Curry summarizing the (then) forthcoming book of the same title by Curry and Feys (see next review).

J. BARKLEY ROSSER

HASKELL B. CURRY and ROBERT FEYS. ***Combinatory logic. Volume I.*** With two sections by William Craig. Studies in logic and the foundations of mathematics. North-Holland Publishing Company, Amsterdam 1958, xvi + 417 pp.

The authors say in the Preface that "... combinatory logic aspires to be an analysis of the ultimate foundations ..." This is meant in all seriousness, so that the authors not only discuss specialties of combinatory logic in the fullest possible detail, but add extended discussions of other logical doctrines that might be needed for the ultimate foundations. Thus the first two chapters are taken up with such topics as "Philosophy of a Formal System," "Theory of Definition," and so on. It is not until Chapter 5, on page 151, well over a third of the way through the book, that combinators appear. The authors also strive for the ultimate in abstractness and generality in presenting topics, so as to achieve the maximum breadth in applicability.

At best, this would produce a book that is difficult to read. This difficulty is greatly increased by a sort of casualness on the part of the authors. We shall cite some instances.

On page 14 we find "0", with or without primes, used as a name of an ob; that this is the intended usage is confirmed by a statement on page 21. It is then indicated that "0 = 0 is an elementary statement." This is loose terminology, and leaves one in doubt whether "0 = 0" is a name of the ob ⌜Q00⌝, or whether (as is done on page 241) it is to be taken as synonymous with "⊢Q00" (this is the authors' terminology). On page 28, the authors digress for a paragraph to remark that there are cases where one must be careful about quotation marks to avoid trouble but that such cases will not arise in the present work and so they have permitted themselves "a certain looseness in regard to these matters." It would have been helpful if this *caveat* had been inserted earlier, on page XVI in conjunction with the paragraph explaining the use of quotation marks. However, it still did not resolve the ambiguity noted.

The authors spend eleven pages, 94–104 inclusive, on painstaking proofs about substitution. These proofs suffer a crucial lacuna because of the failure to prove that the notation uniquely indicates the succession of steps used in building up a complex expression from its constituents. Such proofs present no major difficulty, but they are at least as difficult as the proofs of the substitution theorems which are presented in such elaborate detail, and which use implicitly at crucial points the unproved results about the unique denotation of the notation.

The longest chapter in the book has on its second page (p. 278) the statement, "In this chapter we shall study the basic theory of functionality, as defined in §8C." The complete definition, as given in §8C (see page 264), is, "If the F-obs are restricted as stated here, and certain technical restrictions are maintained, we shall speak of the *basic* theory of functionality; ..." Presumably one is to learn what the technical restrictions are by reading the proofs carefully, to see what hypotheses are used besides those stated.

On the whole, it would appear that the inexperienced reader will find the treatment inadequate at many important points, whereas the experienced reader will find it unduly detailed in many spots without necessarily finding it definitive at such spots.

Combinatory logic in the specialized sense remains a constituent of logic worthy of study, not only for the light it sheds on the use of variables but because it may serve other uses. In particular, there are some indications that it may be applicable to the programming of computers. The present book contains much distracting detail that is probably irrelevant for such specialized