Quantification and Modality

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Appearance

- The modal operators are like disguised quantifiers.
  Fitting & Mendelsohn 1998:108
- Why?
  1. Accessibility relations of modals with possible worlds are stated using quantifiers.
  2. De re/de dicto interpretations depend on relative scope of a modal and a quantifier, analogous to the interpretations that arise when two quantifiers are present.
  3. Some modal operators (e.g. temporals like ‘always’ and ‘sometimes’) are in fact quantifiers.
- It [is] no accident that the quantifiers look very much like modals. Koslow 1992: 312
- Why?
  – Because universal and existential quantifiers, in general, are modals, not just in special cases, e.g. when they range over temporal entities.

Reality

A. Universal and existential quantifiers are modal operators, as Koslow contends.
B. Not all modal operators are quantifiers.

- In order to demonstrate A and B, some background is needed.
- I start with an account of the notion of an implication structure.

Implication structures

- An implication structure *I* consists of a set *S* (propositions or other elements) and an implication relation $\Rightarrow$ over them.
- The relation $\Rightarrow$ satisfies the following axioms, where $A_1, \ldots, A_n, B, B_1, \ldots, B_m, C$ are in *S*:
  - Projection: $A_1, \ldots, A_n \Rightarrow A_k$ ($1 \le k \le n$)
  - Simplification: If $A_1, \ldots, A_i, A_i, \ldots A_n \Rightarrow B$, then $A_1, \ldots, A_i, \ldots A_n \Rightarrow B$ ($1 \le i \le n$)
  - Permutation: If $A_1, \ldots, A_n \Rightarrow B$, then $A_{f(1)}, \ldots, A_{f(n)} \Rightarrow B$ for any permutation *f* of $\{1, \ldots n\}$.
  - Cut: If $A_1, \ldots, A_n \Rightarrow B$ and $B, B_1, \ldots, B_m \Rightarrow C$, then $A_1, \ldots, A_n, B_1, \ldots, B_m \Rightarrow C$.

Additional properties of the implication relation $\Rightarrow$

- In $A_1, \ldots, A_n \Rightarrow B$, the A_is are the premises (or antecedents), and *B* is the conclusion (or consequence).
  - Reflexivity: $A \Rightarrow A$ (follows from Projection)
  - Transitivity: If $A \Rightarrow B$ and $B \Rightarrow C$, then $A \Rightarrow C$ (follows from Cut)
  - Antisymmetry: If $A \Rightarrow B$ and $B \Rightarrow A$, then $A \Leftrightarrow B$.
  - Monotonicity: If $A_1, \ldots, A_n \Rightarrow B$, then $A_1, \ldots, A_n, C \Rightarrow B$ (follows from Simplification and Cut).

Dual of an implication structure

- If $I = \langle S, \Rightarrow \rangle$ is an implication structure, then its dual $\hat{I} = \langle S, \hat{\Rightarrow} \rangle$ is also an implication structure, where:

  $A_1, \ldots A_n \hat{\Rightarrow} B$ iff for every *T* in *S*, if $A_1 \Rightarrow T, \ldots, A_n \Rightarrow T$, then $B \Rightarrow T$.
- In the single premise case, this condition reduces to:

  $A \hat{\Rightarrow} B$ iff $B \Rightarrow A$.

A simple implication structure I_4 with four elements

- Let $I_4 = \langle S_4, \Rightarrow \rangle$, where $\Rightarrow$ is a truth-preserving relation and $S_4 = \{a, b, c, d\}$ such that:
  a: 'it's hot' ⇔ ¬b
  b: 'it's cold' ⇔ ¬a
  c: 'it's hot and it's cold' ⇔ (a∧¬a) ⇔ ⊥ (bottom)
  d: 'it's hot or it's cold' ⇔ (a∨¬a) ⇔ T (top)
- Notation:
  ¬ negation
  ∧ conjunction
  ∨ disjunction

Diagram of I_4, omitting reflexive arcs and arcs derivable from transitivity

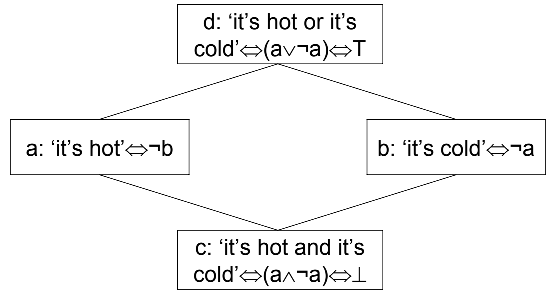
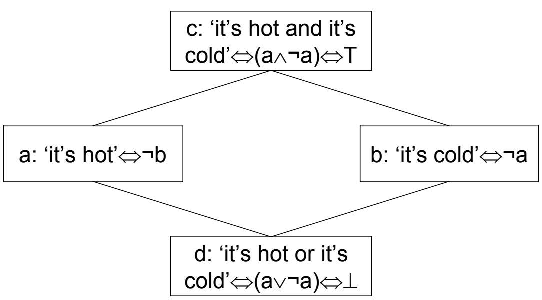


Diagram of $\hat{I}_4 = \langle S_4, \hat{\Rightarrow} \rangle$, dual of I_4, in which $\hat{\Rightarrow}$ preserves falsity



Modal operators

- Next, I define what a modal operator is, following Koslow 1992, providing a purely 'structural' account, without using the notion of 'possible worlds'. There are two types.
  - Necessity, represented by □ 'box'.
  - Possibility, represented by ◊ 'diamond'.

Defining properties of a □ modal

- □ is a unary operator that preserves implication in an implication structure I.
  If $A_1, \ldots, A_n \Rightarrow B$, then $\Box A_1, \ldots, \Box A_n \Rightarrow \Box B$, for all $A_1, \ldots, A_n, B$ in S.
- □ fails to preserve implication in the dual structure $\hat{I}$.
  There are $A_1, \ldots, A_n, B$ in S such that $A_1, \ldots, A_n \hat{\Rightarrow} B$, but $\Box A_1, \ldots, \Box A_n \hat{\Rightarrow} \Box B$ fails.

Distribution of □ with ∧ and ∨

- For the structures of interest to us, the following distribution principles are consequences of the defining properties of □.
  DAX ("*distribution of and with box*")
  $\Box A \wedge \Box B \Leftrightarrow \Box(A \wedge B)$ for all A, B in S.
  NOX ("*nondistribution of or with box* ")
  $\Box(A \vee B) \Rightarrow \Box A \vee \Box B$ fails for some A, B in S.
  - Selecting $B \Leftrightarrow \neg A$, the premise is generally true and the consequent false.

Defining properties of a $\Diamond$ modal

- $\Diamond$ is a unary operator that fails to preserve entailment in *I*.

  There are $A_1, ..., A_n, B$ in *S* such that $A_1, ..., A_n \Rightarrow B$, but $\Diamond A_1, ..., \Diamond A_n \Rightarrow \Diamond B$ fails.
- $\Diamond$ preserves entailment in $\hat{I}$.

  If $A_1, ..., A_n \hat{\Rightarrow} B$, then $\Diamond A_1, ..., \Diamond A_n \hat{\Rightarrow} \Diamond B$, for all $A_1, ..., A_n, B$ in *S*.
  - In $\hat{I}$, $\Diamond$ is a necessity modal and $\Box$ is a possibility modal.

Distribution of $\Diamond$ with $\wedge$ and $\vee$

- For the structures of interest to us, the following distribution principles are consequences of the defining properties of $\Diamond$.

  NAD ("<u>n</u>ondistribution of <u>a</u>nd with diamon<u>d</u>")

  $\Diamond A \wedge \Diamond B \Rightarrow \Diamond(A \wedge B)$ fails for some *A*, *B* in *S*.
  - See comment on NOX.

  DOD ("<u>d</u>istribution of <u>o</u>r with diamon<u>d</u>")

  $\Diamond(A \vee B) \Leftrightarrow \Diamond A \vee \Diamond B$ for all *A*, *B* in *S*.

Interdefinability of $\Box$ and $\Diamond$

- $\Box$ and $\Diamond$ are interdefinable using negation in all implication structures of interest here; i.e. for all *A* in *S*:

  $\Box A \Leftrightarrow \neg\Diamond\neg A$

  $\Diamond A \Leftrightarrow \neg\Box\neg A$

Alethic and epistemic modality in I_4

- Next, I illustrate two types of modality in the structure I_4, alethic (only analytic statements are necessary) and epistemic (both analytic and known synthetic statements are necessary).
  - Let $\Box_\alpha$ and $\Diamond_\alpha$ represent alethic necessity and possibility.

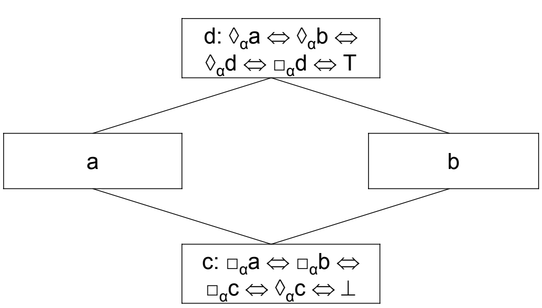
    $\Box_\alpha$T = T, otherwise $\Box_\alpha A = \bot$

    $\Diamond_\alpha \bot = \bot$, otherwise $\Diamond_\alpha A$ = T
  - Let $\Box_\varepsilon$ and $\Diamond_\varepsilon$ represent epistemic necessity and possibility.

    $\Box_\varepsilon A = A$ if *A* is known, otherwise $\Box_\varepsilon A = \bot$

    $\Diamond_\varepsilon A$ = T if *A* is known, otherwise $\Diamond_\varepsilon A = A$

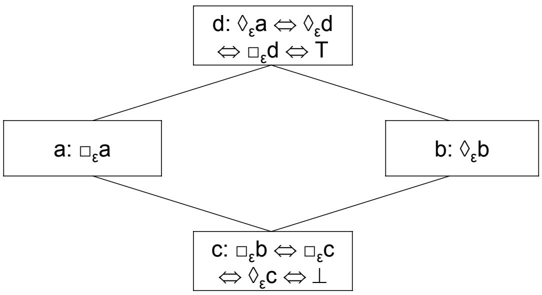
Diagram of alethic modality in I_4



Alethic modals in I_4 have the requisite distribution properties

- DAX
  - $\Box_\alpha a \wedge \Box_\alpha b \Leftrightarrow \Box_\alpha(a \wedge b) \Leftrightarrow c$.
  - Similarly for all other pairs of elements of S_4.
- NOX
  - $\Box_\alpha(a \vee b) \Leftrightarrow d$; $\Box_\alpha a \vee \Box_\alpha b \Leftrightarrow c$; $d \Rightarrow c$ fails.
- NAD
  - $\Diamond_\alpha a \wedge \Diamond_\alpha b \Leftrightarrow d$; $\Diamond_\alpha(a \wedge b) \Leftrightarrow c$; $d \Rightarrow c$ fails.
- DOD
  - $\Diamond_\alpha(a \vee b) \Leftrightarrow \Diamond_\alpha a \vee \Diamond_\alpha b \Leftrightarrow d$.
  - Similarly for all other pairs of elements of S_4.

Diagram of epistemic modality in I_4



Epistemic modals in I_4 also have the requisite distribution properties

- DAX
  - $\Box_\varepsilon a \wedge \Box_\varepsilon b \Leftrightarrow \Box_\varepsilon(a \wedge b) \Leftrightarrow c$.
  - Similarly for all other pairs of elements of S_4.
- NOX
  - $\Box_\varepsilon(a \vee b) \Leftrightarrow d$; $\Box_\varepsilon a \vee \Box_\varepsilon b \Leftrightarrow a$; $d \Rightarrow a$ fails.
- NAD
  - $\Diamond_\varepsilon a \wedge \Diamond_\varepsilon b \Leftrightarrow b$; $\Diamond_\varepsilon(a \wedge b) \Leftrightarrow c$; $b \Rightarrow c$ fails.
- DOD
  - $\Diamond_\varepsilon(a \vee b) \Leftrightarrow \Diamond_\varepsilon a \vee \Diamond_\varepsilon b \Leftrightarrow d$.
  - Similarly for all other pairs of elements of S_4.

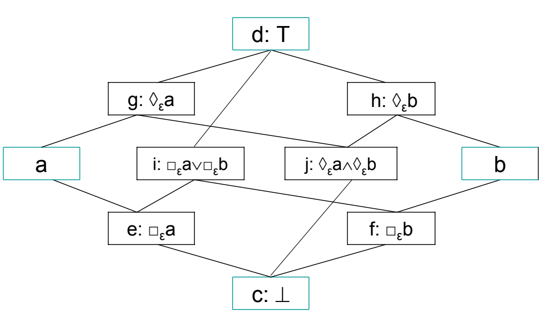
Additional properties of alethic and epistemic modals in I_4

- For all A in S_4:
  - $\Box_\alpha A \Rightarrow \Box_\varepsilon A \Rightarrow A \Rightarrow \Diamond_\varepsilon A \Rightarrow \Diamond_\alpha A$
  - $\Box_\alpha \Box_\varepsilon A \Leftrightarrow \Box_\varepsilon \Box_\alpha A \Leftrightarrow \Box_\alpha A$
  - $\Diamond_\alpha \Diamond_\varepsilon A \Leftrightarrow \Diamond_\varepsilon \Diamond_\alpha A \Leftrightarrow \Diamond_\alpha A$
  - $\Diamond \Box A \Rightarrow \Box \Diamond A$ (regardless of subscript on $\Box$ and $\Diamond$)
  - $\Diamond_\alpha A \Rightarrow \Box_\alpha \Diamond_\alpha A$ and $A \Rightarrow \Box_\alpha \Diamond_\alpha A$
- However:
  - $\Diamond_\varepsilon A \Rightarrow \Box_\varepsilon \Diamond_\varepsilon A$ and $A \Rightarrow \Box_\varepsilon \Diamond_\varepsilon A$ fail for $A = b$.

A linguistically realistic epistemic structure I_{10}

- Let I_{10} be the structure $<S_{10}, \Rightarrow>$, where $S_{10} = S_4 \cup$ {e, f, g, h, i, j}, such that:
  - e: 'it must be hot' $\Leftrightarrow \Box_\varepsilon a$
  - f: 'it must be cold' $\Leftrightarrow \Box_\varepsilon b$
  - g: 'it might be hot' $\Leftrightarrow \Diamond_\varepsilon a$
  - h: 'it might be cold' $\Leftrightarrow \Diamond_\varepsilon b$
  - i: 'it must be hot or it must be cold' $\Leftrightarrow \Box_\varepsilon a \vee \Box_\varepsilon b$
  - j: 'it might be hot and it might be cold' $\Leftrightarrow \Diamond_\varepsilon a \vee \Diamond_\varepsilon b$

Diagram of epistemic modality in I_{10}



Additional properties of epistemic modals in I_{10}

- $\Box_\varepsilon e \Leftrightarrow \Box_\varepsilon f \Leftrightarrow \Box_\varepsilon i \Leftrightarrow \Box_\varepsilon j \Leftrightarrow c$; $\Box_\varepsilon g \Leftrightarrow a$; $\Box_\varepsilon h \Leftrightarrow b$
- $\Diamond_\varepsilon g \Leftrightarrow \Diamond_\varepsilon h \Leftrightarrow \Diamond_\varepsilon i \Leftrightarrow \Diamond_\varepsilon j \Leftrightarrow d$; $\Diamond_\varepsilon e \Leftrightarrow a$; $\Diamond_\varepsilon f \Leftrightarrow b$
- Consequently for all A in S_{10}:
  - $\Diamond_\varepsilon \Box_\varepsilon A \Rightarrow \Box_\varepsilon \Diamond_\varepsilon A$
  - $\Box_\varepsilon \Box_\varepsilon A \Rightarrow \Box_\varepsilon A$, but not conversely
  - $\Diamond_\varepsilon A \Rightarrow \Diamond_\varepsilon \Diamond_\varepsilon A$, but not conversely
  - $A \Rightarrow \Box_\varepsilon \Diamond_\varepsilon A$ (recall that this fails in I_4)
- However:
  - $\Diamond_\varepsilon A \Rightarrow \Box_\varepsilon \Diamond_\varepsilon A$ fails for A = a and A = b.

Finally we get to quantifiers! But first, ...

- The analysis of quantifiers requires the use of extended implication structures I = $<E, \Pi, S, \Rightarrow>$, where E is a set of entities, Π a set of predicates (open sentences), S a set of (closed) sentences, and $\Rightarrow$ a truth-preserving implication relation. For ease of presentation, I consider only one-place predicates.

Predicate implication and universal quantification $\forall$ 'for all'

- Predicate implication $\Rightarrow^*$ is defined in terms of sentential implication as follows.

  For all P_1, ..., P_n, Q in Π, $P_1, ..., P_n \Rightarrow^* Q$ iff $P_1(e_1), ..., P_n(e_n) \Rightarrow Q(f)$ for all choices of $e_1, ..., e_n, f$ in E.
- That is (where $\forall P_1$ represents $\forall x P_1 x$, etc.):

  $P_1, ..., P_n \Rightarrow^* Q$ iff $\forall P_1, ..., \forall P_n \Rightarrow \forall Q$

Unifying predicate and sentential implication

- Extending sentential implication to also cover predicate implication (so that both open and closed sentences can appear in the same implication, and so that the implication relation over open sentences is the same as that over closed ones), we have:

  If $P_1, ..., P_n \Rightarrow Q$ then $\forall P_1, ..., \forall P_n \Rightarrow \forall Q$
- That is, the universal quantifier $\forall$ distributes over implication, so is a necessity modal operator.

Confirming that $\forall$ is a necessity modal

- $\forall$ satisfies DAX. For all predicates P, Q in Π:

  $\forall P \wedge \forall Q \Leftrightarrow \forall(P \wedge Q)$
  - This is a well known property of universal quantification.
- $\forall$ satisfies NOX. For some predicates P, Q in Π:

  $\forall(P \vee Q) \Rightarrow \forall P \vee \forall Q$ fails.
  - If neither P nor Q holds for every member of E but their disjunction does, then the premise is true and the consequent false.

The existential quantifier $\exists$ 'for some' is a possibility modal

- $\exists$ satisfies NAD. There are predicates P, Q in Π such that:

  $\exists P \wedge \exists Q \Rightarrow \exists(P \wedge Q)$ fails.
  - If P and Q hold for different members of E, then the premise is true and the consequent false.
- $\exists$ satisfies DOD. For all predicates P, Q in Π:

  $\exists(P \vee Q) \Leftrightarrow \exists P \vee \exists Q$
  - This is a well known property of existential quantification.

Interdefinability of $\forall$ and $\exists$

- $\forall$ and $\exists$ are interdefinable using negation, just like $\Box$ and $\Diamond$. For all P in Π:

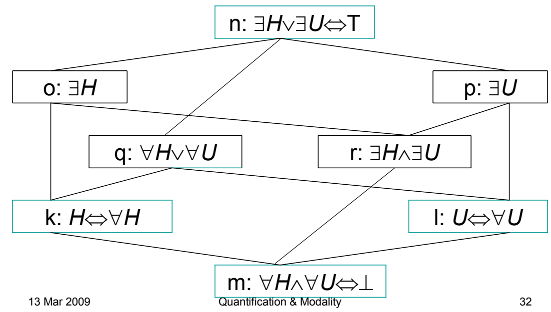
  $\forall P \Leftrightarrow \neg\exists\neg P$

  $\exists P \Leftrightarrow \neg\forall\neg P$

A simple quantificational structure I_8

- Let I_8 be the extended structure $<E, \Pi_2, S_8, \Rightarrow>$, where E is a plural set of people, $\Pi_2 = \{H$ 'happy', U 'unhappy'$\}$, $S_8 = \{k, l, m, n, o, p, q, r\}$, $\Rightarrow$ is a truth-preserving relation, and:
  - k: $H \Leftrightarrow \forall H$ 'everyone is happy'
  - l: $U \Leftrightarrow \forall U$ 'everyone is unhappy'
  - m: $\forall H \wedge \forall U \Leftrightarrow \bot$ 'everyone is happy and everyone is unhappy'
  - n: $\exists H \vee \exists U \Leftrightarrow T$ 'someone is happy or someone is unhappy'
  - o: $\exists H$ 'someone is happy'
  - p: $\exists U$ 'someone is unhappy'
  - q: $\forall H \vee \forall U$ 'everyone is happy or everyone is unhappy'
  - r: $\exists H \wedge \exists U$ 'someone is happy and someone is unhappy'

Diagram of quantificational modality in I_8



Reality check A

A. The principle that universal and existential quantifiers are modal operators is now established.

- Quantified modal logic is simply modal logic, which includes the modal quantifiers $\forall$, $\exists$, and (as shown below) others in its purview.
- In natural languages, modals are expressed not only by tense, mood and aspect markers and adverbials, as Fitting & Mendelsohn 1998: 2 contend, but also by quantifiers that reduce the placedness of predicates.
- *Teaser question*: Might there also be modals that increase the placedness of predicates?

Reality check B

B. The principle that not all modal operators are quantifiers is established by observing that the alethic and epistemic modals we have considered are not quantifiers.

- However, the appearances noted by Fitting & Mendelsohn are not deceiving.
  - Classification of modals according to properties of their accessibility relations can be accounted for without using possible-world models; see Koslow 1992: chapters 34-35.
  - De re/de dicto interactions follow from the general modal law $\Diamond\Box \Rightarrow \Box\Diamond$, from which we obtain $\exists\Box_\varepsilon \Rightarrow \Box_\varepsilon\exists$ and $\Diamond_\varepsilon\forall \Rightarrow \forall\Diamond_\varepsilon$ as special cases; also, for multiple quantification, $\exists\forall \Rightarrow \forall\exists$.

Other modal quantifiers

- $\forall$ and $\exists$ are not the only modal quantifiers.
- Let $I_z = <E_z, \Pi, S_z, \Rightarrow>$ be an extended implication structure in which E_z is the set of positive integers and Π is a set of one-place predicates over E_z.
- Then $\forall/f$ and $\exists\infty$ are modal quantifiers in I_z where:

  $\forall/f$ represents 'for all but at most finitely many members of E_z'.

  $\exists\infty P$ represents 'for infinitely many members of E_z'.

$\forall/f$ is a necessity modal

- $\forall/f$ satisfies DAX.

  $\forall/fP \wedge \forall/fQ \Leftrightarrow \forall/f(P \wedge Q)$ for all P, Q in Π.
  - This equivalence follows from the fact that the union of finite sets is finite.
- $\forall/f$ satisfies NOX.

  $\forall/f(P \vee Q) \Rightarrow \forall/fP \vee \forall/fQ$ fails for some P, Q in Π.
  - If $Q = \neg P$, where P and Q each hold for infinitely many positive integers, then the premise is true and the consequent false.

$\exists\infty$ is a possibility modal

- $\exists\infty$ satisfies NAD.

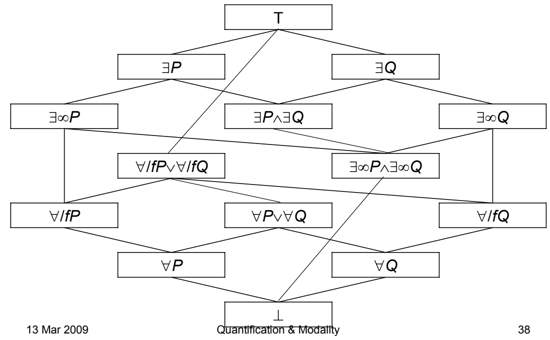
  $\exists\infty P \wedge \exists\infty Q \Rightarrow \exists\infty(P \wedge Q)$ fails for some P, Q in Π.

  – Again, If $Q = \neg P$, where P and Q each hold for infinitely many positive integers, then the premise is true and the consequent false.
- $\exists\infty$ satisfies DOD.

  $\exists\infty(P \vee Q) \Leftrightarrow \exists\infty P \vee \exists\infty Q$ for all P, Q in Π.

  – This equivalence also follows from the fact that the union of finite sets is finite.

Diagram of I_z including $\forall$, $\forall/f$, $\exists$, $\exists\infty$



Other quantifiers are not modal

- Let $I_I = \langle E_I, \Pi, S_I, \Rightarrow\rangle$ be an extended implication structure in which E_I = is large. If E_I is finite, let n = card(E_I), $k = n/2$ for even n, and $k=(n-1)/2$ for odd n.
- Then none of the following quantifiers are modal:

  $\forall/1$ 'for all but at most 1' $\exists 2$ 'for at least 2'
  ... ...
  $\forall/k$ 'for all but at most k' $\exists k+1$ 'for at least k+1'
  ... ...
  $\forall/w$ 'for most' = 'for all but at most a few'
  $\exists m$ 'for at least many'

Implicational properties of these other quantifiers

- Let $\forall^*$ represent $\forall/1$, ... $\forall/k$, ..., $\forall/w$ and let $\exists^*$ represent $\exists 2$, ... $\exists k+1$, ... $\exists m$. Then:

  $\forall^*$ and $\exists^*$ preserve one-premise implications but fail to preserve multi-premise implications.

Why $\forall^*$ and $\exists^*$ preserve one-premise implications

- If $P \Rightarrow Q$, then $\{x|Px\} \subseteq \{y|Qy\}$. Consequently, $\forall^*P \Rightarrow \forall^*Q$ and $\exists^*P \Rightarrow \exists^*Q$.
- Counter-models showing that $\forall/1$ and $\exists 2$ fail to preserve multi-premise implications are provided in the next slide; similar models can be provided for all the other members of $\forall^*$ and $\exists^*$.

Why $\forall^*$ and $\exists^*$ fail to preserve multi-premise implications

- $P, Q \Rightarrow P \wedge Q$ for all P, Q in Π. However for some P, Q in Π:

  $\forall/1P, \forall/1Q \Rightarrow \forall/1(P \wedge Q)$ fails.

  $\exists 2P, \exists 2Q \Rightarrow \exists 2(P \wedge Q)$ fails.

  – In the first case, if P and Q fail to hold for different single members of E only, then the premise is true but the consequent is false.

  – In the second case, if P and Q hold for distinct pairs of members of E only, then the premise is true but the consequent is false.

Quasimodal operators

- Let us call unary operators that preserve one-premise implications but not multi-premise ones quasimodals.
- Symbolizing quasimodals by ✡, we have:
  1. If $A \Rightarrow B$, then ✡$A \Rightarrow$ ✡B for all A, B in S.
  2. However, there are A_1, ... A_n, B in S such that A_1, ... $A_n \Rightarrow B$, but ✡A_1, ... ✡$A_n \Rightarrow$ ✡B fails.
- Similarly for $\hat{\Rightarrow}$.

Quasimodal quantifiers

- ∀* and ∃* are all quasimodals, more precisely, quasimodal quantifiers, that satisfy the requirement:

  If $P \Rightarrow Q$, then ✡$P \Rightarrow$ ✡Q for all P, Q in Π. However, there are P_1, ... P_n, Q in Π such that P_1, ... $P_n \Rightarrow Q$, but ✡P_1, ... ✡$P_n \Rightarrow$ ✡Q fails.

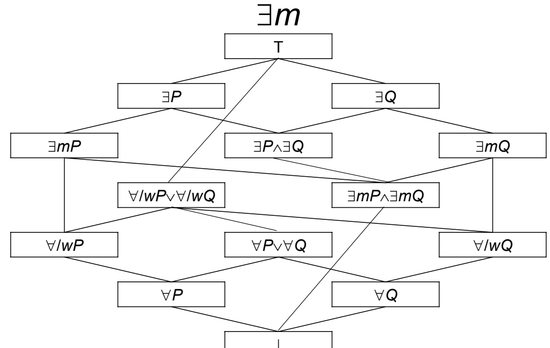
Quasinecessity and quasipossibility

- Quasimodals, like modals, come in interdefinable pairs. For example:

  ∀/1 ⇔ ¬∃2¬ ∀/*w* ⇔ ¬∃*m*¬

  ∃2 ⇔ ¬∀/1¬ ∃*m* ⇔ ¬∀/*w*¬
- Let us call the implicationally stronger of these pairs (e.g. ∀/1, ∀/*w*) a quasinecessity (or quasiuniversal), symbolized ▽, and the weaker (e.g. ∃2, ∃*m*) a quasipossibility (or quasiexistential), symbolized △.

Diagram of I_l including ∀, ∀/*w*, ∃, ∃*m*



Comparing the modals ∀/*f* and ∃∞ with the quasimodals ∀/*w* and ∃*m*

- The diagram of I_l including ∀, ∀/*w*, ∃, ∃*m* has exactly the same form as the diagram of I_z including ∀, ∀/*f*, ∃, ∃∞, even though ∀/*f* and ∃∞ are modal and ∀/*w* and ∃*m* are quasimodal.
- In I_z, in which all these operators are defined, the implicational subhierarchy:

  ∀/*f* ⇒ ∀/*w* ⇒ ∃*m* ⇒ ∃∞

  lies within the hierarchy

  ∀ ⇒ ∀/1 ⇒ ... ∀/*k* ... [ ] ... ∃*k*+1 ⇒ ... ∃2 ⇒ ∃

  in the position indicated by [ ].

Non-quantifier quasimodals

- In a large enough implication structure in which the epistemic modals $\Box_\varepsilon$ and $\Diamond_\varepsilon$ are defined, epistemic quasinecessity and quasipossibility operators $\triangledown_\varepsilon$ and $\triangle_\varepsilon$ can also be defined such that for all A in S:

  $\Box_\varepsilon A \Rightarrow \triangledown_\varepsilon A \Rightarrow \triangle_\varepsilon A \Rightarrow \Diamond_\varepsilon A$

  analogous to ∀* and ∃* in relation to ∀ and ∃.
  - In such a structure $\triangledown_\varepsilon$ may be expressed by ‘probably’ and $\triangle_\varepsilon$ as ‘not improbably’.

'Probably' and 'not improbably' are quasimodals, not modals

- The operator $\triangledown_\varepsilon$ 'probably' is a quasimodal, since it preserves one-premise implications, but not multi-premise ones, in a rich enough epistemic structure.
  - The answer to Fitting & Mendelsohn's 1998: 3, Exercise 1.1.1.1: Is 'probably' a modal? is ... quasipositive!
- Also, since $\triangle_\varepsilon$ 'not improbably' $\Leftrightarrow \neg\triangledown_\varepsilon\neg$, it too is a quasimodal in that structure.
- Finally, since $\triangledown_\varepsilon \Rightarrow \triangle_\varepsilon$, the former is a quasinecessity and the latter a quasipossibility.

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