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The Logic of Reciprocity[1]

1. Goals, Limits, and Conclusions of this Study

The semantic properties of expressions containing reciprocal elements and their antecedents have figured prominently in a number of recent discussions of linguistic theory.[2] Nevertheless, these properties are not particularly well understood, despite such recent pioneering efforts as Dougherty (1970; 1971; 1974) and Fiengo and Lasnik (1973).[3] The difficulty of the subject matter suggests the strategy of first examining the structurally most simple expressions in which both a reciprocal element and its antecedent appear. These are sentences of the form (1), in which A denotes a set of cardinality $\geqq 2$, R is a relation on $A \times A$,[4] and r is a reciprocal element, for example one of the phrases *each other* and *one another*.[5]

(1) A R r

Such expressions we call *elementary reciprocal sentences* (ERSs). Our examination of ERSs is further limited to the study of the contributions made by r and by its relation to A to the truth conditions (TCs) of those sentences.[6] We assume at the outset of our investigation that these contributions are the same for all ERSs; that is, that the TCs for

[1] Versions of this article were read at MIT, University of Reading, and Universiteit van Amsterdam. Much of the work reported on here was carried out while I was a Fulbright-Hays Senior Lecturer at Instituut voor Algemene Taalwetenschap, Rijksuniversiteit Utrecht, and an earlier version appears in *Utrecht Working Papers in Linguistics* 3, 85–114, 1977. I especially thank Suzanne Duggan, Robert Fiengo, Fredric Katz, Jerrold Katz, David Nash, Stanley Peters, and William Stewart for helpful comments on and criticisms of this material.

[2] See, for example, Chomsky (1973, 238ff., esp. 239, fn. 18 and 255–256, fn. 34; 1975, 101–104, 109–110, 140–143, 148–152).

[3] Cf. Chomsky (1975, 150): "Suppose we also have a *reciprocal rule* that gives the meaning of structures of the form . . . *NP* . . . *one another* . . . , when the two phrases in question are 'anaphorically related.' This rule is not without its complications."

[4] We assume, for simplicity, that tense and aspect are incorporated into R.

[5] We ignore here the prescriptivist use condition that calls for the use of *each other* if the cardinality of A is 2 and of *one another* if it is greater than 2, and use mainly *one another* throughout, following what appears to be a trend in current English usage (Bernstein (1965, 155)). There may be other differences, in different dialects of English, in the use and even the logic of *each other* and of *one another,* but these, too, will be ignored.

[6] By *the truth condition of an ERS* we mean the condition that must be satisfied in order for an assertion of the proposition literally expressed by the ERS to be true. We assume that the statement of TCs either is an aspect of semantic representation, or derives from semantic representations by interpretive principles.

ERSs can be stated generally, with A and R appearing as schematic letters, standing for arbitrary plural sets and binary relations, respectively. This is by no means a necessary assumption, but it is one that we will be able to justify in the course of the investigation.[7]

We begin by considering six not unlikely potential schematic representations of the TCs for ERSs, and we reach a tentative conclusion as to which one best represents the TCs for ERSs. Then we go on to consider a class of elementary sentences that properly includes the class of ERSs, namely those of the form (2), in which both A and B denote (not necessarily disjoint) sets of cardinality $\geqq 2$, and R is a relation on $A \times B$.

(2) A R B

Such sentences we call *elementary plural relational sentences* (EPRSs). We show that our tentative formulation of the TC-schema for ERSs follows compositionally from the TC-schema for EPRSs, upon substitution of r for B, thus justifying both our initial assumption about the uniformity of the contribution of r and of its relation to A to the interpretation of ERSs and our specific formulation of the TC-schema for ERSs. Moreover, given the assumptions of the standard theory of the organization of generative grammar, it follows that r must occur as an element (possibly as an idiomatic phrase, as in English) in the underlying representations of ERSs.

Next we compare the TC-schema for ERSs with that of a disjoint class of elementary sentences, of the form (3), in which A denotes a set of cardinality $\geqq 2$ and R is a symmetric relation on $A \times A$.

(3) A R

Sentences like these have an affinity with explicitly reciprocal sentences that contain a symmetric relation, as Gleitman (1965), among others, has observed, and for that reason we call them *elementary covertly reciprocal sentences* (ECRSs). We show that the TC-schema for ECRSs is, however, distinct from that of corresponding ERSs, thus removing the possibility, under the assumptions of the standard theory, of deriving ECRSs and their corresponding ERSs from common underlying sources.

Following that demonstration, we consider the problem of the interpretation of a certain class of ERSs, discussed by Fiengo and Lasnik (1973), the assertions of which are deemed true in situations in which the appropriate substitution-instances of the TC-schema for ERSs are not satisfied. We conclude that this discrepancy follows from a use condition that is appropriate to that class, so that the general form of the TC-schema for ERSs can be maintained.

[7] Fiengo and Lasnik (1973), for example, contend that the logical structure of reciprocal sentences depends on certain general semantic properties of R. For a criticism of their contention, see fn. 10.

Finally, we consider the problem of extending the results of this article to a wider class of reciprocal sentences, so as to provide a general account of the logic of reciprocity.

2. Some Possible Schematic Representations of the TCs for ERSs

Various possible schematic representations of the TCs for ERSs have been suggested in the literature, and many others are imaginable. In this section we give six logically distinct, yet not implausible, schematizations and show how they are logically related.

(A) *Strong Reciprocity* (SR) (the *each-the-other* relation of Fiengo and Lasnik (1973)).

$$(\forall x,y \in A)(x \neq y \rightarrow xRy)$$

(B) *Partitioned Strong Reciprocity* (PSR) (the *reciprocal* relation of Fiengo and Lasnik (1973); the *distinct subsets* relation of Dougherty (1974)). Let $A = A_1 \cup \ldots \cup A_n$ and $(\forall i, j,\ 1 \leqq i, j \leqq n)(i \neq j \rightarrow A_i \cap A_j = \phi)$ and $(\forall k,\ 1 \leqq k \leqq n)$ (card $A_k \geqq 2$).

$$(\forall i,\ 1 \leqq i \leqq n)(\forall x,y \in A_i)(x \neq y \rightarrow xRy)$$

(C) *Symmetric Reciprocity* (SmR) (the *unrestricted subsets* relation of Dougherty (1974)).

$$(\forall x \in A)(\exists y \in A)(x \neq y \wedge xRy \wedge yRx)$$

(D) *Intermediate Reciprocity* (IR) (the *connectedness* relation of standard logic).

$$(\forall x,y \in A)\{x \neq y \rightarrow [xRy \vee (\exists n > 0)(\exists z_1, \ldots, z_n \in A)(xRz_1 \wedge \ldots \wedge z_nRy)]\}$$

(E) *Partitioned Intermediate Reciprocity* (PIR). Let A be partitioned into subsets as in (B).

$$(\forall i,\ 1 \leqq i \leqq n)(\forall x,y \in A_i)\{x \neq y \rightarrow [xRy \vee (\exists n > 0)(\exists z_1, \ldots, z_n \in A)(xRz_1 \wedge \ldots \wedge z_nRy)]\}$$

(F) *Weak Reciprocity* (WR).

$$(\forall x \in A)(\exists y,z \in A)(x \neq y \wedge x \neq z \wedge xRy \wedge zRx)$$

These TCs are logically related as shown in (4).

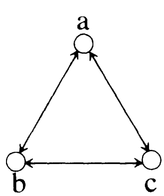
(4) a. (card A = 2) → (SR = PSR = SmR = IR = PIR = WR)
b. (card A = 3) → (SR = PSR → SmR → IR = PIR = WR)

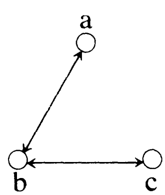
$$\text{c. (card A = 4)} \rightarrow \begin{tikzcd} & \text{PSR} \arrow[r] & \text{SmR} \arrow[dr] & \\ (\text{SR} \arrow[ur] \arrow[dr] & & & \text{PIR} = \text{WR}) \\ & \text{IR} \arrow[urr] & & \end{tikzcd}$$

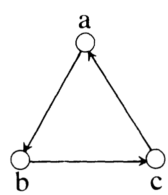
$$\text{d. (card A} \geqq 5) \rightarrow \begin{tikzcd} & \text{PSR} \arrow[r] & \text{SmR} \arrow[dr] & \\ (\text{SR} \arrow[ur] \arrow[dr] & & & \text{PIR} \rightarrow \text{WR}) \\ & \text{IR} \arrow[urr] & & \end{tikzcd}$$

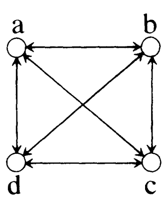
These relations can be illustrated by diagrams in which small circles indicate members of the set A and directed arrows indicate the relation R. Thus, $\underset{a}{\circ}\!\rightarrow\!\underset{b}{\circ}$ indicates that aRb holds, but not bRa; $\underset{a}{\circ}\!\leftarrow\!\underset{b}{\circ}$ indicates that bRa holds, but not aRb; $\underset{a}{\circ}\!\leftrightarrow\!\underset{b}{\circ}$ indicates that both aRb and bRa hold; and finally $\underset{a}{\circ}\quad\underset{b}{\circ}$ indicates that neither aRb nor bRa holds.

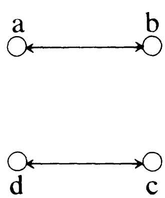
(5) a. 

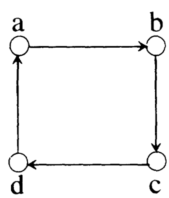
b. i. 

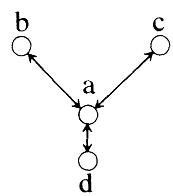
ii. 

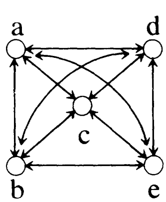
iii. 

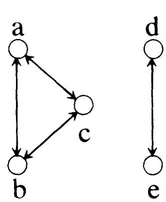
c. i. 

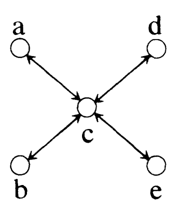
ii. 

iii. 

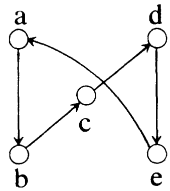
iv. 

d. i. 

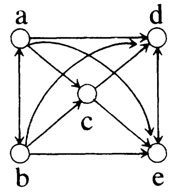
ii. 

iii. 

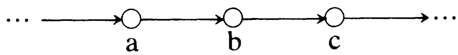
iv.



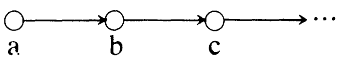
v.



e. i.



ii.



The diagrams in (5) may be used to establish the relations stated in (4) as follows.

To establish (4a), we observe that the situation-type diagrammed in (5a) is the only one involving sets of cardinality 2 that satisfies any of the conditions given in (A)–(F), and that all six are satisfied by it.

To establish (4b), we observe (i) that the situation-type diagrammed in (5bi) is the only one involving sets of cardinality 3 that satisfies SR and PSR (it also, of course, satisfies the other four TCs); (ii) that the situation-type in (5bii) satisfies the remaining four TCs; (iii) that the situation-type in (5biii) satisfies IR, PIR, and WR, but not SmR; and (iv) that there are no situation-types for sets of cardinality 3 that distinguish between SR and PSR or among IR, PIR, and WR, or that introduce other distinctions among the six TCs.

To establish (4c), we observe (i) that the situation-type diagrammed in (5ci) is the only one involving sets of cardinality 4 that satisfies SR (it also satisfies the other five TCs); (ii) that the situation-type diagrammed in (5cii) satisfies PSR, SmR, PIR, and WR, but not IR; (iii) that the situation-type in (5ciii) satisfies IR, PIR, and WR, but not PSR and SmR; (iv) that the situation-type in (5civ) satisfies SmR, IR, PIR, and WR, but not PSR; and (v) that there are no situation-types for sets of cardinality 4 that distinguish between PIR and WR, or that introduce other distinctions among the six TCs.

Finally, to establish (4d), we observe (i) that the situation-type diagrammed in (5di) is the only one involving sets of cardinality 5 that satisfies SR (it also satisfies the other five TCs); (ii) that the situation-type diagrammed in (5dii) satisfies PSR, SmR, PIR, and WR, but not IR; (iii) that the situation-type in (5diii) satisfies SmR, IR, PIR, and WR, but not PSR; (iv) that the situation-type in (5div) satisfies IR, PIR, and WR, but not PSR and SmR; (v) that the situation-type in (5dv) satisfies WR, but not PSR, SmR, IR, and PIR; and (vi) that there are no other situation-types for sets of cardinality 5 or greater that introduce other distinctions among the six TCs. In addition, WR and only

WR can be satisfied by an asymmetric, disconnected[8] relation R on an infinite set A for which the relation is not founded,[9] as (5e) illustrates.

3. Evaluation of the Various TC-Schemata for ERSs

Fiengo and Lasnik (1973) argue convincingly that ERSs may be satisfied by situation-types of the sort diagrammed in (5cii), and hence that neither SR nor IR provides the correct characterization of the TC-schema of ERSs in general.[10] However, contrary to the judgment of Fiengo and Lasnik, we find that PSR must also be rejected, since there are ERSs whose TCs are satisfied by situation-types like (5bii) and (5civ), which (as we have already seen) do not satisfy PSR. Thus, consider an ERS like (6).

(6) The men and the woman flirted with one another.

If, in (5bii), we take a to be *the woman* and b and c to be *the men*; and if, in (5civ), we take a to be *the woman* and b, c, and d to be *the men*; and if, in both diagrams, we take the relation R to be *flirted with,* then we see that those situation-types satisfy the TC of (6).[11]

[8] We say that a relation R on $A \times A$, where A is of cardinality $\geqq 2$, is disconnected just in case (i) holds.

(i) $(\forall x \in A)(\sim\exists y \in A)\{(x \neq y \wedge xRy) \rightarrow [yRx \vee (\exists n > 0)(\exists z_1, \ldots, z_n \in A)(yRz_1 \wedge \ldots \wedge z_nRx)]\}$

If a relation R is neither connected (as defined in (D)) nor disconnected, then we say that it is nonconnected.

[9] For the formal definition of *foundedness,* see Quine (1969, 141).

[10] Fiengo and Lasnik (1973) actually contend that SR is the TC-schema for ERSs if the relation R is stative, and that PSR is the TC-schema for ERSs otherwise. Against the former claim, consider the example (i), cited by Chomsky (1975, 150), which is an ERS whose relation is stative, but which is satisfied by situations of the type diagrammed in (5cii).

(i) John's grandparents hate one another.

This situation-type, as we have seen, is not one that satisfies SR.

Thus, there is no reason to believe that separate TC-schemata must be given for ERSs depending on whether R is stative or nonstative. Fiengo and Lasnik also discuss another class of ERSs for which a special TC-schema might have to be set up. We take up this class of cases below in section 7, and argue there that an alternative analysis of this class can be given such that a single TC-schema that is applicable to all ERSs can be maintained. If this argument is correct, then we will have shown that every objection that has been given so far against setting up more than one TC-schema for ERSs can be met.

[11] The subject noun phrase in (6) is deliberately chosen to highlight the fact that situations like (5bii), (5ciii), and (5diii) can satisfy the TC-schema for ERSs. But even for an ERS like (i), in which, say, A refers to three men and a woman, we see that the situation-type (5ciii) satisfies the TC of that sentence.

(i) They flirted with one another.

From the discussion in Dougherty (1974, 14), however, we can infer that he disagrees with the judgments just rendered, and that he has in mind a TC-schema for ERSs that is distinct from any of those proposed here. In his discussion, Dougherty considers examples like (ii) and (iii) and claims that (ii) cannot truly describe a situation in which heterosexual relations only are involved (actually, Dougherty hedges this claim by saying that such an interpretation, if made, would be "forced"), but that (iii) can (throughout, we assume that the names *John, Bill, Tom,* and *Sam* denote males and that the names *Sue* and *Mary* denote females).

(ii) John, Bill, Tom, Sam, and Mary had relations with each other.
(iii) John, Bill, Tom, Sue, and Mary had relations with each other.

We can sharpen this discussion somewhat by changing Dougherty's examples to (ii′) and (iii′).

(ii′) John, Bill, Tom, Sam, and Mary had only heterosexual relations with each other.
(iii′) John, Bill, Tom, Sue, and Mary had only heterosexual relations with each other.

The next candidate in line is SmR, and it too must be rejected, since it does not allow for the possibility that an ERS may be satisfied by situation-types like those diagrammed in (5biii) and (5ciii). Consider, for example, sentences like (7)–(8).[12]

(7) They displaced one another.
(8) They scratched one another's backs.

Clearly, the assertions made by these sentences are true in appropriate situations of the types diagrammed in (5biii) and (5ciii). Since these situation-types do not satisfy SmR, only PIR and WR remain as potentially correct characterizations of the TC-schema for ERSs.

As we showed in section 2, PIR and WR are equivalent for sets A of cardinality 4 or less, but not for sets of cardinality 5 or greater. The simplest situation-type that distinguishes between PIR and WR is given in (5dv), but it is of such complexity as to make it extremely difficult to determine by introspection or other means whether the assertion of an ERS is true or false in such a situation. We might, however, hope to get clear judgments for ERSs whose relations necessarily hold either symmetrically or asymmetrically for any given pair of members of A. I know of no lexical item in English that has this logical property, but any composite relation of the form *are at least as X as* does. Suppose, then, we let A consist of five individuals, of whom two weigh 50 kg each, one weighs 60 kg, and two weigh 70 kg each. We now ask: in this situation, which is of type (5dv), is the assertion of the ERS (9) true or false?

(9) They are at least as heavy as one another.

Unfortunately, sentences like (9) are felt to be bizarre independent of the truth or falsity of the assertions that they make (for example, even if the five individuals weigh exactly the same amount, so that (9) makes an indisputedly true assertion, the sentence still seems strange). In my judgment, however, if the bizarreness of (9) is somehow set aside, then its assertion is felt to be true in the situation just described. If this judgment is correct, then PIR cannot be the TC-schema for ERSs, leaving only WR as a

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Dougherty's position entails the claim that the literal assertion of (ii′) cannot be true (or that such a use would be "forced"), but that the assertion of (iii′) can be. Now, none of the TC-schemata defined in section 2 distinguishes between (ii′) and (iii′); according to SR and PSR, both would necessarily make false assertions, while according to SmR, IR, PIR, and WR, neither would. So, what criterion could Dougherty have in mind for distinguishing between them? He offers nothing precise, and it is difficult to see what such a criterion, if made precise, could be. If, for example, the criterion requires that no one individual in A both relate and be related to significantly more than any other individual in A, how would such significance be nonarbitrarily defined? For example, would the assertion of a sentence like (iv) be necessarily false under this criterion?

(iv) Those thirty-seven men and six women had only heterosexual relations with each other.

In the face of this difficulty, it would seem more appropriate to adopt the position that the assertions of (ii′) and (iv) are not necessarily false, and that the assertion of (ii) could be true under the condition that only heterosexual relations are involved.

[12] Example (8) is not, strictly speaking, an ERS, but this detail does not affect the validity of the argument in any way. To see why, see section 8 below.

candidate of those that we have considered. But clearly, more evidence is required before it can be decided with certainty that WR should be adopted as the TC-schema for ERSs.[13]

From our discussion in section 2, we see that we might obtain further evidence from consideration of ERSs with an asymmetric, disconnected relation R. If PIR is the TC-schema for ERSs, then the assertions of all such sentences are false; if WR is the TC-schema for ERSs, then the assertions of such sentences are true if the set A is infinite and not founded with respect to R, and false otherwise. Accordingly, compare sentences (10)–(12).

(10) The numbers from one to four succeed one another.
(11) The natural numbers succeed one another.
(12) The integers succeed one another.

The relation *succeed,* as used in these examples, is asymmetric and disconnected. In (10), A is a finite set, and, in accordance with both PIR and WR, the assertion of that sentence is false. In (11), A is an infinite set, but since the set is founded with respect to the relation *succeed* (there is a member of the set, the number zero, that succeeds no other member of the set), the assertion of that sentence, too, is false, in accordance with both PIR and WR.[14] Finally, in (12), A is an infinite set that is not founded with respect to the relation *succeed* (since the integers range over all negative and nonnegative numbers, there is no integer that succeeds or is succeeded by no other integer). Hence, according to PIR, the assertion of that sentence is false; but according to WR, it is true. Once again, in my judgment, the prediction made by WR is correct, providing further evidence that WR is to be preferred to PIR as the TC-schema for ERSs. However, the evidence still must be considered insufficient to establish the case with certainty.

Since no other type of empirical evidence is forthcoming, we must look elsewhere for considerations that bear on the problem of choosing between PIR and WR as the schema that correctly represents the TCs for ERSs.

4. The Relation of the Logic of Reciprocity to the Logic of Plurality

An ERS is a special type of elementary sentence with plural subject and plural object, that has the general form given in (2) (repeated here), and that we called an *elementary plural relational sentence* (EPRS).

(2) A R B

[13] In addition, both PIR and WR predict that (9) makes a false statement in case the weights of the five individuals are 50 kg, 50 kg, 60 kg, 60 kg, and 70 kg.

[14] However, there may be conditions of use in which the assertions made by sentences like (10) and (11) are true; see below, section 7, in particular fn. 31.

As a first approximation to the schematic representation of the TCs for EPRSs, consider (G).[15]

(G) $(\forall x \in A)(\exists y \in B)(xRy) \wedge (\forall w \in B)(\exists z \in A)(zRw)$

According to (G), for the literal assertion of a sentence of the form (2) to be true, each member of the set *A* must bear the relation *R* to some member of the set *B*, and each member of the set *B* must have the relation *R* borne to it by some member of the set *A*. For example, consider the EPRS (13).

(13) The women released the prisoners.

If (G) is the TC-schema for EPRSs, then (13) entails both (14) and (15).

(14) Each woman released some prisoner.
(15) Each prisoner was released by some woman.

Surely no stronger entailments are possible; for example, (13) certainly does not entail (16), showing that the existential quantifiers in (G) cannot be replaced by universal ones.

(16) Each woman released each prisoner.

The possibility remains, however, that (G) is too strong. For example, suppose that exactly one woman in the set denoted by *the women* was not involved in the releasing of the prisoners at all. Could we not still say that the assertion of (13) is true? Certainly (13) could be used to describe a situation in which in fact all but one of the women was involved in releasing the prisoners, but then we should say that (13) is not literally being asserted. Moreover, this fact of usage should not obscure the semantic fact that (13) is not entailed by (17).[16]

(17) All but one of the women released the prisoners.

Hence, the literal assertion of (13) is not true in case exactly one (or more than one) woman in the set denoted by *the women* was not involved in releasing the prisoners.[17]

[15] The requirement that A and B denote sets of cardinality $\geqq 2$ is not essential, and may be dropped; that is, the TC-schema for EPRSs generalizes to sentences in which either the subject set or the object set or both have exactly one member. This is obviously a desirable result (cf. Bierwisch (1970, 30–31)). We keep the requirement here simply to bring out the parallelism between EPRSs and ERSs.

[16] If (17) entails (13), then by parity of reasoning (i) should entail (17), which entails (13).

(i) All but two of the women released the prisoners.

Proceeding in this fashion, we obtain (ii), which also entails (13).

(ii) All but all but one of the women released the prisoners. (i.e., Exactly one of the women released the prisoners.)

But this is absurd. Hence (17) cannot entail (13).

[17] Note that we are not claiming that a false sentence can be used to make a true assertion. Sentences (and the propositions they express) are not the sorts of things that are true or false; only their assertions are. To take another example, consider the following sentence.

There remains, however, the possibility that two or more of the women may have acted in the release of one or more of the prisoners in such a way that none of those women can be said to have individually released any of the prisoners. Indeed, all of the women may have acted in this way. In such cases, (13) remains true, but (14) is false. Thus, (13) entails (14) only under the special condition that the women acted as individuals, and not as members of some larger group of women. In the next section, we show how to reformulate (G) to handle the more general cases. In this section, we retain the simplifying assumption that the sets A and B that appear in EPRSs have only individuals (unit subsets) as members.

Given that (G) is the TC-schema for EPRSs, and given that an ERS is a special type of EPRS, in which r appears in the place of B, it should be possible to deduce the TC-schema for ERSs from (G) together with the specific contribution of r to the interpretation of ERSs. It is evident that one of the consequences of the presence of r in ERSs is that the object set B is taken to be identical to the subject set A. But that is not the only consequence, since if a reflexive element (e.g. *themselves*) appears as object in an EPRS, the same consequence obtains, and yet ERSs are logically distinct from EPRSs with reflexive objects.[18] In addition, the presence of r in ERSs has the consequence that the relation is specifically understood as holding between distinct members of the set A.

We obtain the TC-schema for ERSs from the TC-schema for EPRSs, therefore, as follows. By setting set A equal to set B in (G), and identifying variables, we obtain (H).[19]

(H) $(\forall x \in A)(\exists y,z \in A)(xRy \wedge zRx)$

(i) The women considered the consequences of their action.

When uttered, this sentence may be used to assert that the women in question considered the salient consequences of their action, and if they did, then that assertion is true. It remains true even if they did not consider any of the infinitely many nonsalient logical consequences of their action. If the proposition literally expressed by (i) entails that the women considered consequences of their action other than the salient ones, then the assertion of *that* proposition in this situation would be false. By hypothesis, however, it is not that proposition that was asserted, but rather the one whose assertion is true.

A final comment. We take sentences like (13) to be logically equivalent to sentences in which *either* the subject or the object noun phrase is explicitly quantified by *all*. The addition of this word in this way, while not affecting the TCs of the resulting sentences, does affect how they can be used. In particular, it is harder to use such sentences under conditions in which the assertion of the proposition literally expressed is false.

[18] Moreover, consider sentences like (i), under the interpretation that the subject and object sets have identical members.

(i) The women released their mothers' daughters.

This sentence turns out to be logically distinct from both (ii) and (iii).

(ii) The women released themselves.
(iii) The women released one another.

To see this, let $A = \{a,b,c\}$, and consider the following situation: a released a, b released c, and c released b. Then (i) is true, but both (ii) and (iii) are false.

[19] (H) provides the TC-schema for EPRSs like example (i) of fn. 18, under the interpretation given there. It also provides the TC-schema for French EPRSs whose object is the element *se* (and also of similar EPRSs

Then, by adding the requirement that the relation specifically hold between distinct members of A, we obtain (I).

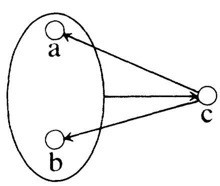
(I) $(\forall x \in A)(\exists y,z \in A)(x \neq y \wedge x \neq z \wedge xRy \wedge zRx)$

But (I) = (F) = the TC-schema we have called WR. That is, WR follows as the TC-schema for ERSs from the TC-schema for EPRSs by substitution of the interpretation of the reciprocal element for the object phrase.[20] Therefore, our assumption that there is a single TC-schema for ERSs is justified, and we can conclude, despite the inconclusiveness of the empirical evidence for selecting WR as the TC-schema for ERSs, that indeed WR is the TC-schema for ERSs. Moreover, from the fact that the interpretation of ERSs follows compositionally from the interpretation of EPRSs, we can conclude that the contribution of *r* is like that of any lexical item or phrasal idiom, and hence that it should be treated as a lexical item or phrasal idiom in the grammars of natural languages. We shall have more to say on this matter below in section 8.

5. Generalization of the TC-Schemata for ERSs and EPRSs

In the preceding section, we pointed out that the statement (G) of the TC-schema for EPRSs failed to handle situations in which nonunit subsets of the sets *A* and *B* bear the relation *R* to one another. The statement (F) of the TC-schema for ERSs has the same inadequacy. To see this, let $A = \{a,b,c\}$, let *R* be the relation *released,* and consider the situation-type diagrammed in (18), in which *(a and b)Rc* entails neither *aRb* nor *bRc*.

(18)



in other Romance languages). Accordingly, the French sentence (i) is logically equivalent to (i) of fn. 18 (under the given interpretation), not to (ii) or (iii) of fn. 18.

(i) Les femmes se sont libérées.

The French literal equivalents of (ii) and (iii) of fn. 18 are respectively (ii) and (iii) below.

(ii) Les femmes se sont libérées elles-mêmes.
(iii) Les femmes se sont libérées l'un à l'autre.

Even in singular sentences of the type (iv) in French, *se* is not to be interpreted as a specifically reflexive element.

(iv) La femme s'est libérée.

Rather, the reflexive interpretation of (iv) is a logical consequence of (H), where the cardinality of *A* is 1 (see fn. 15).

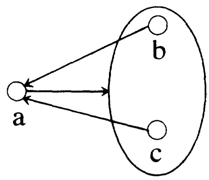
[20] The fact that the cardinality of *A* is $\geqq 2$ in ERSs is a logical consequence of WR and also does not have to be specifically indicated.

This situation-type fails to satisfy WR; nevertheless, the assertion of the ERS (19) is true.

(19) They released one another.

Similarly, consider a situation-type like the one diagrammed in (20), in which *aR*(*b and c*) entails neither *aRb* nor *aRc*.

(20)



It is somewhat more difficult to find convincing English examples that illustrate this situation-type, although perhaps (21) will be found persuasive.

(21) They endorsed one another.

If it is possible that one can endorse two or more persons as a group (for example, because of their combined virtues) without endorsing any one person individually, then (21) is satisfied by the situation-type (20), which in turn fails to satisfy WR.

However, it is not difficult to generalize WR to express a TC-schema that situations of the type (18) and (20) do satisfy, and that therefore can be accepted as the TC-schema for ERSs. Essentially, we must provide that each member of the set *A* be a member of a subset of *A* that bears the relation *R* to some nonnull subset of *A* not containing that member, and that each member of *A* is a member of a subset of *A* that has the relation *R* borne to it by some nonnull subset of *A* not containing that member. Formally we express this TC-schema as (J); it will be observed that (J) is a generalization of the notion of WR to relations between subsets of *A*.

(J) *Weak Reciprocity for Subsets* (WRS).

$$(\forall x \in A)(\exists X_1, X_2, Y \neq \phi, Z \neq \phi \subseteqq A)(x \in X_1 \wedge x \in X_2 \wedge x \notin Y \wedge x \notin Z \wedge X_1 R Y \wedge Z R X_2)$$

WRS reduces to WR when X_1, X_2, Y, and Z are taken to be unit subsets of A, as the reader can verify.

If WR is the TC-schema for ERSs, then it must be derivable from the TC-schema for EPRSs in the manner discussed above in section 4. Accordingly, the TC-schema for EPRSs must be stated as in (K).

(K) $$(\forall x \in A)(\exists X \subseteqq A, Y \neq \phi \subseteqq B)(x \in X \wedge XRY) \wedge (\forall w \in B)(\exists W \subseteqq B, Z \neq \phi \subseteqq A)(w \in W \wedge ZRW)$$

(K) converts to (J) for ERSs in the same way that (G) converts to (F). Moreover, given a relation R for which either *(a and b)Rc* does not entail *aRb* or *bRc,* or *aR(b and c)* does not entail *aRb* or *aRc*, propositions of the type expressed by (14) or (15) do not follow from a given EPRS. We may therefore conclude that (K) is the TC-schema for EPRSs and that (J) is the TC-schema for ERSs.

6. Covert Reciprocity

In this section, we consider the logical relation between ERSs containing symmetric relations, such as (22), and their counterparts in which no reciprocal element appears, such as (23).

(22) They are similar to one another.
(23) They are similar.

This relation has received considerable attention in the literature; see in particular Gleitman (1965); Lakoff and Peters (1969); Stockwell, Schachter, and Partee (1973, 298–305, 311–314); Fiengo and Lasnik (1973). In section 1, we called sentences like (23), which are of the general form (3), where A denotes a set of cardinality $\geqq 2$ and R is a symmetric relation on $A \times A$, *elementary covertly reciprocal sentences* (ECRSs).

(3) A R

An interesting discussion of the logic of ECRSs can be found in Leonard and Goodman (1940), in which it is demonstrated that SR (and hence also all of the other TC-schemata for ERSs given in section 2, including WR) is too weak a TC-schema for such sentences.[21] In particular, they criticize Carnap for having supposed that "a class of things each member of which is similar to each other [member thereof] is a class of things which are all similar" (p. 53). They contend, in other words, that the TC of a sentence like (23) is more stringent than even that provided by SR. Their argument is a straightforward constructive one, as follows (p. 51).

> Suppose we have as elements a set of three columns, each colored with three bands, as pictured in [(24)], and suppose that the relation S is such that 'xSy' means that in some one band—lower, middle, or upper—the two entities x and y are identically colored. . . . It is clear . . . that we may have three columns, like the ones pictured, such that *aSb, bSc,* and *aSc* [and by symmetry of *S,* also *bSa, cSb,* and *cSa*/DTL], even though all three columns have no single color in any one band.

[(24)]

K	K	J
G	L	L
M	H	M
a	*b*	*c*

The capital letters represent distinct shades of color.

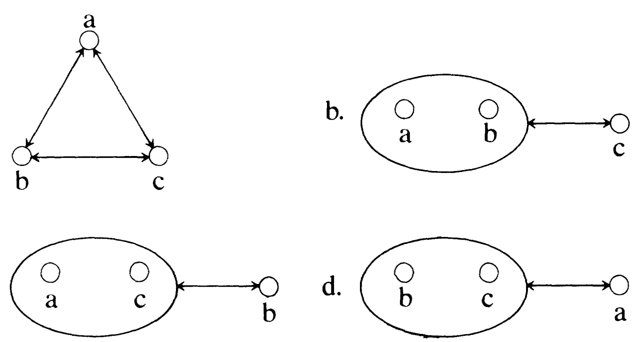
[21] I thank Fredric Katz for bringing this article to my attention.

If we construe the relation S to be like *are similar to,* as Leonard and Goodman's choice of schematic letter and subsequent discussion suggest, we see that the situation diagrammed in (24) satisfies the TC for the ERS (22), but not that of the corresponding ECRS (23).[22] In order for the TC of (23) to be satisfied, all of the colors in some one band would have to be identical. However, if we generalize the definition of SR to relations between subsets of A, just as we generalized that of WR in section 5, we see that the resulting notion does provide a correct characterization of the TC-schema for ECRSs like (23).[23]

(L) *Strong Reciprocity for Subsets* (SRS).
$(\forall x \in A)(\forall X,Y \neq \phi \subseteqq A)[(x \in X \wedge X \cap Y = \phi) \rightarrow XRY]$

For sets A of cardinality 3, the four situation-types diagrammed in (25) must hold together, in order for SRS to be satisfied.[24]

(25) a.



Not every ECRS has a corresponding ERS; for example, (26) does not.[25]

(26) They assembled.

The converse is also true; there are ERSs with symmetric relations that have no ECRS counterpart, for example (27) and (28).[26]

[22] Indeed, (22) would be satisfied by (24) even if the color in the lower band of c were G instead of M, since WR would still be satisfied.

[23] This definition is, in essence, a translation into Boolean notation of the characterization given by Leonard and Goodman (1940) in terms of the "calculus of individuals". The stipulation that the intersection of the sets X and Y should be null is to enable the definition to apply correctly to ECRSs with irreflexive, symmetric relations like *meet.*

[24] Depending on the relation R involved, the propositions satisfied by one of the situation-types in (25), say (25a), may entail all of those satisfied by the others.

[25] The ERS that appears to correspond to (26), namely (i), in fact does not, since the relation expressed by *assembled* in (i) is not the same as the relation expressed by *assembled* in (26).

(i) They assembled one another.

[26] The sentence that appears to be an ECRS corresponding to (27), namely (i), in fact is not an ECRS.

(i) They are married.

Example (i) is not a relational sentence at all, but rather one in which a property (expressed by *are married*)

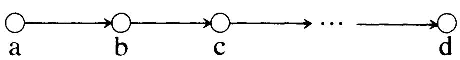
(27) They resemble one another.
(28) They are married to one another.

Therefore, we find no reason for grammatically relating ERSs like (22) and ECRSs like (23). They are distinct both syntactically and semantically. The former contain overt reciprocal elements; the latter do not. The TC-schema of the former is provided by WRS; the TC-schema of the latter by SRS.[27]

7. A Problem Concerning ERSs with Asymmetric, Disconnected Relations

As we observed in section 2, situations of the type diagrammed in (5ei) satisfy WR. In such situations, R may be an asymmetric, disconnected relation on $A \times A$, and A must be an infinite set that is not founded with respect to R. However, as Fiengo and Lasnik (1973) observe, many ERSs with such relations are also used to make true assertions in situations in which A is a finite set. For example, (29) can be used to make a true assertion in a situation of the type diagrammed in (30), a situation-type that satisfies neither WR nor WRS.

(29) The plates are stacked on top of one another.

(30) 

To see that (29) is not a totally isolated case, consider the following examples.

(31) The boxes are nested inside one another.
(32) The children are lined up behind one another.
(33) The children are lined up to the right/left of one another.
(34) The guests followed one another (into the room).

Each of these sentences can be used to make true assertions in situations of the type (30). But not every ERS with an asymmetric, disconnected relation can be so used, for example (35) and (36).

(35) The guests preceded one another (into the room).
(36) They are fathers of one another./They are one another's fathers.

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is predicated of the individuals that comprise its subject. The interpretation of (i) is neutral with respect to the question of whether any of those individuals are married to one another.

[27] Accordingly, there is absolutely no reason to derive (23) from the structure underlying (22) by a rule of reciprocal deletion, contrary to a proposal by Gleitman (1965). In addition, we find that the following pair of sentences have distinct TCs.

(i) These books and those books are similar.
(ii) These books are similar to those books.

Example (i) is an ECRS, and therefore has a TC that is a substitution-instance of (L). Example (ii) is an EPRS and has a TC that is a substitution-instance of (K). Moreover, these TCs are satisfied by different classes of situations. Consequently, there is also no reason to derive (ii) from the structure underlying (i) by a rule of conjunct movement, contrary to a proposal by Lakoff and Peters (1969).

Sentences like (35) and (36) cannot be used to make true assertions at all.

Clearly, what is needed is some principle, either grammatical or extragrammatical in nature, that enables one to distinguish ERSs like (29) and (31)–(34) from those like (35)–(36) in their ability to be used to make true assertions in situations of the type (30).[28]

It seems highly unlikely that any principle that can be formulated in terms of a suitably restricted theory of grammar can be found—for example, one that somehow "suspends" the requirements of WRS for ERSs like (29) and (31)–(34), but not for those like (35)–(36); or one that would somehow derive sentences like (29) and (31)–(34) from some other source that does not have to satisfy WRS, but would not so derive (35)–(36).[29] On the other hand, there is some reason to believe that an explanation for this distinction can be found on extragrammatical grounds.

We note that ERSs that can be used to make true assertions in situations of the type (30) all involve spatial or temporal relations that order the elements of the set denoted by *A* in one of the ways listed in (37).[30]

(37) a. from top to bottom
  b. from outside to inside
  c. from front to back
  d. from left to right or from right to left
  e. from earlier to later

However, if the relation is neither spatial nor temporal, or if the ordering is not one of those in (37), then ERSs with an asymmetric, disconnected relation cannot be used to make true assertions in situations of the type (30), for example (35)–(36) above and (38)–(40) (but see fn. 31).

[28] It may be necessary to extend the principle so as to account for the oddity of sentences like (i), in which WRS is satisfied (the set *A* being infinite and the relation not being founded with respect to it).

(i) The integers precede one another.

[29] For example, one might propose to derive (29) from the structure that also underlies (i), and assign it the same TC as (i).

(i) The plates are stacked one on top of the other.

But such an analysis would fail for (34), since the latter has no analogue to (i); moreover, the analysis would have no means to distinguish between (29) and (38), which is the central problem facing us. Hence, such an analysis must be rejected.

[30] The ordering is established as follows. First, find all those elements $a_1, \dots, a_n$ in *A* for which there is no *b* in *A* such that bRa_i, $1 \leqq i \leqq n$. For each such a_i, find an element a_{i_1} in *A* such that $a_i R a_{i_1}$. This establishes a partial ordering (a_i, a_{i_1}) for each a_i. If there is an element a_{i_2} in *A* such that $a_{i_1} R a_{i_2}$, add that element to the appropriate partial ordering. Proceed in this fashion until, for each a_i, a_{i_j} in *A* is found such that there is no *b* in *A* such that $a_{i_j} Rb$. This completes the various partial orderings. This way of establishing the ordering is of sufficient generality to handle cases like (i), which cannot be handled by any simpler technique.

(i) The shoes are heaped on top of one another.

(38) The plates are stacked underneath one another.
(39) The boxes are placed outside one another.
(40) The children are lined up in front of one another.

A particularly interesting contrast is provided by ERSs with the asymmetric, disconnected relation *succeed(ed)*. We have already given an example of one such ERS, in which the set A is finite and in which the relation is understood nontemporally, namely (10), and in accordance with WRS and with (37) it cannot be used to make a true assertion.[31]

(10) The numbers from one to four succeed one another.

Now compare this example with (41), which strikes one not so much as a contradiction, but as a truism.

(41) The monarchs of England succeeded one another (to the throne).

The reason that (41), unlike (10), can be used to make a true assertion is that the relation *succeeded* in it is understood temporally and orders the members of A from earlier to later. Therefore, in accordance with (37e), it can be used to make a true assertion in a situation of the type (30).

I cannot give a precise account of the extragrammatical principle that accounts for the conditions under which ERSs with asymmetric, disconnected relations can be used to make true assertions in situations of the type (30). It would appear that the ordering of elements that is required in order for such sentences to be so used is, however, a natural (or possibly culturally determined) one. We normally stack things one on top of the other rather than the other way around, line up one behind the other rather than one in front of the other, and view time as progressing from earlier to later rather than from later to earlier. Whether we are more likely to put nested boxes one inside the other or the other way around, I cannot say, but we generally perceive them to be nested one inside the other rather than the other way around. Finally, since there is no general preference for arranging things from left to right as opposed to arranging them from right to left, either ordering is acceptable as a basis for the use of sentences like (33) in situations of the type (30).

8. Some Extensions of the Results Obtained So Far

Given the compositionality of the contribution of reciprocal elements to the interpretation of ERSs, we have every reason to believe that the contribution of reciprocal

[31] However, there are people who find that (10) can be used to make a true assertion (to the effect that the numbers from one to four are ordered by the successor relation). For such people, the list in (37) should be expanded to include (f), or possibly even (f′).

f. from smaller to larger
f′. from negative infinity to positive infinity

elements to the interpretation of sentences in general is compositional. Consider, for example, the following sentences, none of which are ERSs but all of which contain a reciprocal element and its antecedent.

(42) The women handed the packages to one another.
(43) The women separated the men from one another.
(44) The women introduced the men to one another.
(45) The women promised the men to watch one another.
(46) The women ordered the men to watch one another.
(47) The women admired the portraits of one another.
(48) The women knew what pleased one another.
(49) The women knew what one another wanted.

In each of these sentences, the reciprocal element has the effect of indicating that a complex, but compositionally determined, relation holds between distinct members (or subsets) of a given plural set, namely the set denoted by the antecedent of the reciprocal element. The problem of determining the antecedent of *r* in (42)–(49) is the familiar problem of control (as discussed, for example, by Chomsky and Lasnik (1977)), but once it is solved, we should have little more trouble determining the TCs of sentences like (42)–(49) than we have had determining the TCs of ERSs.

The solution of the control problem is beyond the scope of this article, and I therefore restrict myself to making only a few remarks concerning it. First, examples like (42)–(46) reveal the already well known fact that the determination of the antecedent of *r* depends in part on the relations that appear in sentences containing *r*. In (42), only the subject phrase can serve as the antecedent of *one another*; in (43), only the object phrase can do so; and in (44), either the subject phrase or the object phrase can do so (that is, (44) is ambiguous with respect to which phrase controls *r*). Similarly, in (45) only the subject phrase of the main clause can serve as the antecedent of *one another,* whereas in (46) only the object phrase of the main clause can do so. For these cases, we can conclude that properties of the verbs *handed, separated, introduced, promised,* and *ordered* determine what can serve as the antecedent of the reciprocal element. Second, examples like (47) show that the complex relation that holds between a reciprocal element and its antecedent can include material that is part of a noun phrase syntactically containing *r*. Finally, examples like (48)–(49) show that the complex relation that holds between a reciprocal element and its antecedent can combine material from an embedded indirect question with the relation of which that indirect question is an object complement. In this last respect, reciprocal elements (at least in English) differ sharply from reflexive ones, since one cannot substitute the reflexive element *themselves* for *one another* in (47)–(48) while preserving grammaticality.[32]

[32] In (48) and (49), moreover, the reciprocal element occurs in a finite clause, under the control of an antecedent lying outside that finite clause, contrary to the "Tensed-S Condition" of Chomsky (1973).

In (49), the reciprocal element even occurs as subject of a finite clause. As far as I know, the only

So far in this discussion, we have described reciprocity as a logical property of sentences that results from the presence of a reciprocal element and its antecedent in those sentences. The obvious alternative, the description of reciprocity as a logical property of relations, like symmetry or transitivity, is inappropriate to English, inasmuch as there are in fact no logically reciprocal relations in English.[33] No English verb, adjective, or predicate noun requires a reciprocal object, and no English verb, adjective, or predicate noun occurs with a plural subject such that the TC of the sentence is a substitution-instance of WRS.

However, there are many languages for which the description of reciprocity as a logical property of relations rather than as a property of sentences arising from the presence of a reciprocal element is appropriate. For example, in Mundari, a Munda language spoken in southern Bihar, India, reciprocal verbs are formed from ordinary transitive verbs by infixing the "reciprocal morpheme" *Vp,* where *V* is the first vowel of the verb, after the first consonant of the verb. Thus, given *nel* 'see', one has *nepel* 'see one another', etc. Reciprocal verbs are grammatically intransitive and require nonsingular (i.e. dual or plural) subjects, and the TCs of reciprocal sentences in Mundari are, as far as I can determine, substitution-instances of WRS. Thus we have sentences like the following in Mundari (hyphens indicate morpheme boundaries within words).[34]

(50) Hon hoṛo nel- ke- d-a-e.
child man see Past TrF S
'The child saw the man.'

(51) Hon-ko hoṛo nel- ke- d-a-ko.
child Pl man see Past TrF S
'The children saw the man.'

(52) Hon-ko hoṛo-ko nel- ke- d-ko-a-ko.
child Pl man Pl see Past TrObF S
'The children saw the men.'

(53) Hon-ko n-ep-el- ke- n-a-ko.
child Pl Rc see Past InF S
'The children saw one another.'

In languages such as Mundari, we may expect to find that certain options that are available in languages that make use of reciprocal elements occurring grammatically as

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context in which a reciprocal element can occur as subject of a finite clause in English is in an indirect question object complement. Not surprisingly, English speakers tend to be rather uncertain about whether a verb that must agree with such an element in number shows singular or plural agreement.

[33] However, if we wished, we could have characterized the relations of ECRSs, such as *are similar* in (23), as logically "strongly reciprocal".

[34] *Tr* is a morpheme indicating that the verb is used transitively; *In* indicates it is used intransitively. *Ob* is a morpheme agreeing in person and number with the object phrase; *F* is a morpheme used to indicate that the sentence occurs independently; and *S* is a morpheme agreeing in person and number with the subject phrase. For further details of Mundari verb morphology, see Langendoen (1967).

noun phrases are not available. For example, the English sentence (46) has no direct translation into Mundari; to express it in Mundari, one would have to resort to a quite elaborate paraphrase.[35]

9. Summary and Conclusion

Reciprocity, as described in this article, is a logical property of sentences that appears to receive direct, simple expression in most, if not all, human languages. However, its interpretation is derivative, arising from the more fundamental logical properties of relational sentences with two plural arguments. We have found, moreover, that any investigation of the interpretation of reciprocal sentences that is limited to an examination only of reciprocal sentences is bound to be inconclusive. Conclusive results can be obtained only if one also examines plural relational sentences, and recognizes how the interpretation of reciprocal sentences derives from the interpretation of plural relational sentences.

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[35] These remarks carry over, with minor changes, to the cross-linguistic description of reflexivity as well.

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