Modals, pseudomodals and quasimodals

Terry Langendoen
Visiting Professor, City U Hong Kong
Professor Emeritus, U Arizona

Linguistics Seminar
Hong Kong U
10 Nov 2008

Some familiar modals in English

- Auxiliary verbs: *must*, *might*, *can*...
- Adverbs: *necessarily, certainly, perhaps*, *possibly, ...*
- Adjectives: *certain* (as in *It is certain that*...), *required*, *permitted*, *able (to)* ...
- Quantifiers: *all*, *each*, *some*, *one*
  - These quantifiers are familiar, but perhaps not as modals.

Interdefinability of modals

- Modals come in pairs, and are generally interdefinable using negation.
  - *necessary = not possible not*; *possible = not necessary not*
  - *required = not permitted not*; *permitted = not required not*
  - *all = not some not*; *some = not all not*

Entailments among modals

- necessary ⇨ possible
- required ⇨ permitted
- all ⇨ some
  - Label the ‘stronger’ one □ (called ‘box’)
  - Label the ‘weaker’ one ◊ (called ‘diamond’)
- □ ⇨ ◊
  - In the examples below, I use initial Greek letters α, β, ... for □ modals and final Greek letters ..., ψ, ω for ◊ modals. They’re also paired, α with ω, etc.

Fundamentals of □

- □ is a unary operator that preserves entailment (or implication): If s_1, ... s_n ⇨ *t*, then □s_1, ,,, □s_n ⇨ □*t*, for all s_1, ... s_n, *t*. A consequence is the equivalence DAB (for “distribution of and with box”):

DAB. □*s*&□*t* ≡ □*s*&*t* for all *s*, *t*

- □ fails to preserve entailment when entailment is reversed (in the ‘dual’). One consequence is NOB (for “nondistribution of or with box ”):

NOB. There are *s*, *t* such that □(*s*|*t)* ⇨ □*s*|□*t* fails.

- To establish NOB, it often works to pick *t* = ¬*s*.

Fundamentals of ◊

- ◊ is a unary operator that does not preserve entailment.
  - There are *s*, *t* and *u* such that if *s*, *t* ⇨ *u*, then ◊*s*, ◊*t* ⇨ ◊*u* fails. One consequence is NAD (for “nondistribution of and with diamond”):

NAD. ◊*s*&◊*t* ⇨ ◊(*s*&*t*) fails for some *s*, *t*.

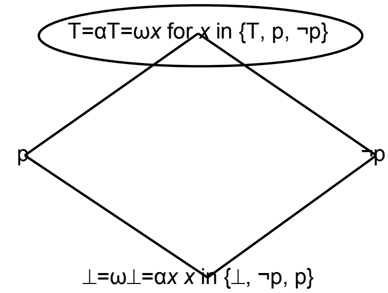
- ◊ preserves entailment in the dual. One consequence is the equivalence in DOD (for “distribution of or with diamond”):

DOD. ◊(*s*|*t*) ⇔ ◊*s*|◊*t* for all *s*, *t*.

Modals and possible world semantics

- The logical properties of modals are usually analyzed in terms of possible world semantics.
  - For example, '*s* is necessary' in a world if and only if *s* is true in all worlds accessible to that world (according to a particular accessibility relation).
- I'll show you another (I hope) simpler way. The next slide shows a toy example of logical necessity and possibility.

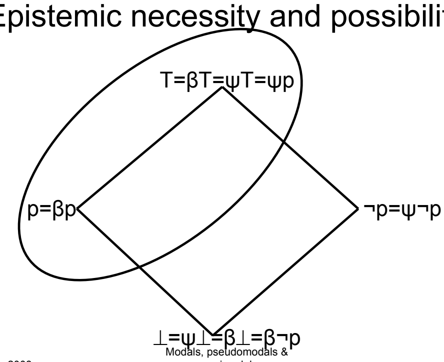
Logical necessity and possibility



How to interpret the diagrams

- The arcs represent the entailment relation between two nodes, upward for the structure under discussion, downward for the dual. Reflexive arcs and arcs derivable from the transitivity of entailment are omitted.
- The ovals enclose a maximally consistent set of □-prefixed elements. In many of these diagrams, there is more than one such set, but only one is marked.

Epistemic necessity and possibility



Not all epistemic operators are modal, some are pseudomodal

- β and ψ are defined as in Table 1. By changing the mappings to Table 2, the resulting epistemic operators B and Ψ no longer satisfy NOB and NAD; both preserve entailment in the original and dual structures. Therefore they are not modal operators, and I refer to them as *pseudomodal* operators.

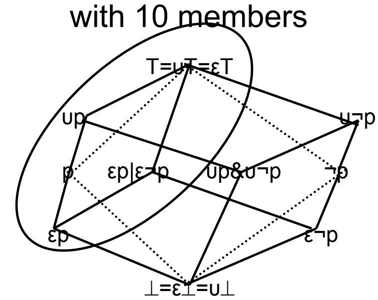
Table 1

	β	ψ
T	T	T
p	*p*	T
¬*p*	⊥	¬*p*
⊥	⊥	⊥

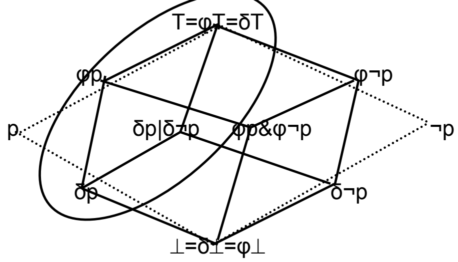
Table 2

	B	Ψ
T	T	T
p	T	T
¬*p*	⊥	⊥
⊥	⊥	⊥

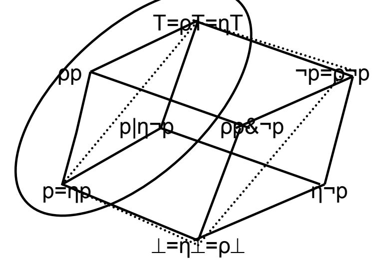
An epistemic modal structure with 10 members



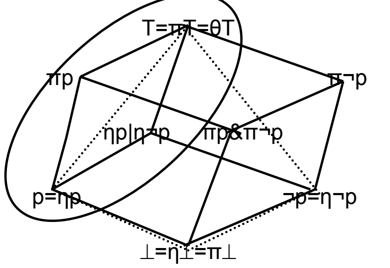
A deontic structure with 10 members (δ = *required*, φ = *permitted*)



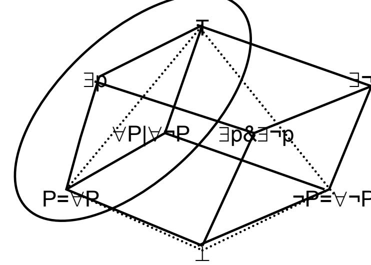
An abilitative structure with 8 members (η =*unable not*, ρ = *able*)



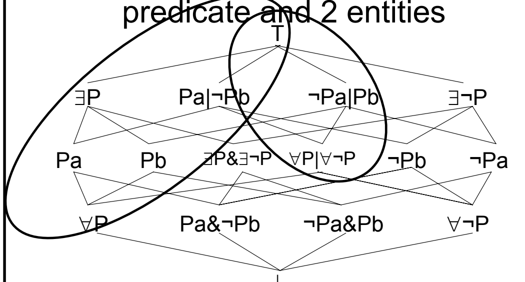
An epistemic structure with 8 members (negation of *p* is not ¬*p*!)



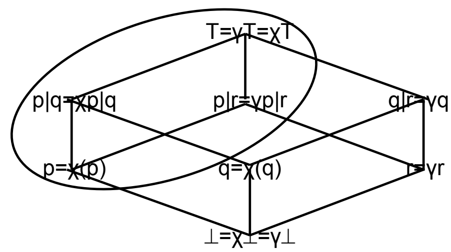
An isomorphic quantificational structure (*P* a 1-place predicate)



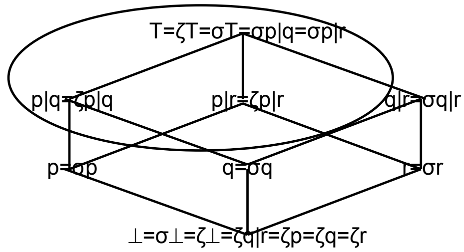
Full quantificational structure with 1 predicate and 2 entities



Another epistemic structure



What sort of operators are ζ and σ?



ζ isn't modal so its dual σ is not the dual of a modal

- ζ fails to preserve entailment in the structure diagrammed in the preceding slide.
  - $p|q$, $p|r \Rightarrow p$, but $\zeta p|q$, $\zeta p|r \Rightarrow \zeta p$ fails, because $\zeta p|q = p|q$, $\zeta p|r = p|r$ and $\zeta p = \bot$.
  - Thus DAB fails, since $(\zeta p|q)\&(\zeta p|r) = p \Rightarrow \zeta((p|q)\&(p|r)) = \bot$ fails.
- ζ and σ are interdefinable using negation, i.e. $\sigma \equiv \neg\zeta\neg$ and $\zeta \equiv \neg\sigma\neg$, so that σ, being the dual of ζ, fails to preserve implication in the dual structure.
  - Thus DOD fails, since $\sigma((p|q)|(p|r)) \Rightarrow (\sigma p|q)|(\sigma p|r)$ fails.

ζ and σ are similar to modals and their duals, so I call them *quasimodals*

- Because ζ and σ fail both DAB and DOD, they satisfy both NOB and NAD, so have those properties in common with □ and ◊.
- Also like □ and ◊, ζ and σ preserve one premise entailments in the original structure.
  - Let ⌂ represents a quasimodal. Then:
- if $s \Rightarrow t$, then ⌂$s \Rightarrow$ ⌂t for all s, t.

Some quasimodals in English

- Quasimodals are found in English, for example the adjective *likely* and its dual *not unlikely*. Both *likely* and *unlikely* fail DAB and DOD, thereby satisfying NOB and NAD, illustrated here for *likely* (represented as *L*) only.
  - Failure of DAB = satisfaction of NAD:
- $Ls\&Lt \Rightarrow L(s\&t)$ fails for some s, t.
  - Failure of DOD = satisfaction of NOB:
- $L(s|t) \Rightarrow Ls|Lt$ fails for some s, t.
  - Also *likely* preserves one-premise entailments:
- If $s \Rightarrow t$ then $Ls \Rightarrow Lt$

A larger quantifier structure

- Consider a quantificational structure with two atomic one-place predicates *P* and *Q* and four entities *a, b, c, d* over which they range.
- Such a structure is too large to diagram, but we can easily reason about quantifiers defined within it. In particular we can define two quasimodal quantifiers, which are interdefinable duals:
  - ∀-1 'for all but at most one'
  - ∃2 'for at least two'

∀-1 and ∃2 fail DAB thereby satisfying NAD

- Let *P* hold for *a*, *b*, *c*, but not *d*, and *Q* hold for *a*, *b*, *d*, but not *c*.
- Then $\forall\text{-}1P\&\forall\text{-}1Q \Rightarrow \forall\text{-}1(P\&Q)$ fails, failing DAB but satisfying NAD.
  - Let *P* hold for *a*, *b* but not *c*, *d*, and *Q* hold for *b*, *c*, but not *a*, *d*.
- Then $\exists 2P\&\exists 2Q \Rightarrow \exists 2(P\&Q)$ fails, failing DAB but satisfying NAD.

∀-1 and ∃2 fail DOD thereby satisfying NOB

- Let *P* hold for *a* but not *b*, *c*, *d*, and *Q* hold for *b* but not *a*, *c*, *d*.
- Then ∃2(*P*|*Q*) ⇨ ∃2*P*|∃2*Q* fails, failing DOD but satisfying NOB.
  - Let *P* hold for *a*, *b* but not *c*, *d*, and *Q* hold for *b*, *c*, but not *a*, *d* (as in the second case on the previous slide).
- Then ∀-1(*P*|*Q*) ⇨ ∀-1*P*|∀-1*Q* fails, failing DOD but satisfying NOB.

Hierarchy of numerical quantifiers

- The quasimodal ∀-1 can be called a quasi-universal quantifier, and its dual ∃2 a quasi-existential one. These are but the first in a series of numerical quantifiers, which can be arranged in an entailment hierarchy (modal and dual modal in orange, quasimodals in turquoise):
- ∀ ⇨ ∀-1⇨ ...∀-*n* ⇨ ∃*n*+1 ⇨ ... ∃2 ⇨ ∃
- If the number of entities in the domain is 2*n*+1, then ∀-*n* ≡ ∃*n*+1, and so is self-dual.

Most *and* many/much *are also quasimodal duals*

- *Most* is a quasi-universal quasimodal that expresses ∀-φ ‘for all but few/little’. Its quasi-existential dual ∃μ *many/much* expresses ‘for many/much’.
  - In addition, *a few/a little* paraphrases *not most*, and *few/little* paraphrases *not many/not much*.

All but finitely many and *infinitely many* are pseudomodals, not quasimodals

- *All but finitely many* (expressing ∀-<∞) and *infinitely many* (expressing ∃∞), are dual quantifiers, the first universal and the second existential, that are pseudomodals, not modals or quasimodals. Both satisfy both DAB and DOD and so preserve entailment in both the original and dual structures.

Full quantifier entailment hierarchy

- Here is the full quantifier entailment hierarchy (modal and dual modal in orange, quasimodals in turquoise, and pseudomodals in lavender):

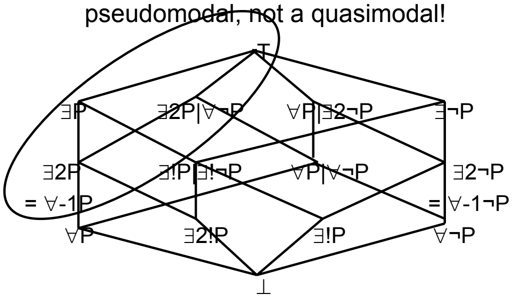
⬀ ∀-*φ* ⇨ ∀-<∞ ⇨ ∃∞ ⇨ ∃μ ⬂

∀ ⇨ ∀-1⇨ ... ∀-*n* ⇨ ∃*n*+1 ⇨ ... ∃2 ⇨ ∃

Acknowledgment

- Thanks especially to Arnold Koslow for the idea that quasimodals preserve entailments with one premise. The approach to modality taken here is generally that of Part IV of his book *A structuralist theory of logic*, Cambridge University Press, 1992.

Postscript: In this quantificational structure with 1 predicate and 3 entities, ∃2 = ∀-1 is a pseudomodal, not a quasimodal!



10 Nov 2008 Modals, pseudomodals & quasimodals 31