LING 501, Fall 2004: Mereology, version 2

This handout replaces the original Mereology handout. It includes the first two sections of the original, but with changes to the second paragraph of the second section.

Mereological implication structures

In the handout on the semantics of connectives, I introduced the notion of a mereological (part-whole) implication structure (MIS), and illustrated it with the Hasse diagram in Figure 1. In this structure, the atomic individuals a, b and c are the singular individuals, and the composite individuals a+b, a+c, b+c, and a+b+c are the plural individuals.

![Figure 1 A simple MIS](image)

Predications over plural individuals

The formula Pa+b represents a one-place predication on the plural individual a+b. Whether Pa+b is equivalent to Pa & Pb depends on P. For example, if P is be light (in weight), then Pa+b entails Pa & Pb, but not conversely. On the other hand, if P is be heavy, then Pa & Pb entails Pa+b, but not conversely. Finally, if P is be friendly, then Pa+b is equivalent to Pa & Pb. If Pa+b entails Pa & Pb, then P is dissective. If Pa & Pb entails Pa+b, then P is cumulative. Thus, be light and be friendly are dissective, and be heavy and be friendly are cumulative. If Pa+b is equivalent to Pa & Pb, then P is cumulodissective.¹

The sentence Alice and Bob are heavy is ambiguous; its meaning can be represented as either Ha+b or as Ha & Hb, indicating that the sentence has two analyses, one simply Ha+b and the other in which a “distributive operator” δ applies to that sentence such that δ(Ha+b) = Ha & Hb.² Presumably then Alice and Bob are friendly also has the analyses Fa+b and δ(Fa+b), these being logically equivalent since be friendly is cumulodissective.

¹ This classification of one-place predicates is due to Nelson Goodman, The structure of appearance, Harvard University Press, 1951.
² See Peter Lasersohn, A semantics for groups and events, Garland, 1990.
Restricted quantification over count terms

The standard FOL “translation” of the sentence (1) is given in (2), where \( R_x \) represents the open sentence ‘\( x \) is a raven’ and \( B_x \) represents \( x \) is black. An alternative translation is one in which the variable is restricted to range over (atomic) individual ravens, as in (3), where \( r \) is such a variable. Similarly, the standard translation of (4) is given in (5). Again restricting the variable to atomic individual ravens, the alternative translation is in (6).

(1) All ravens are black.
(2) \( \forall x [R_x \rightarrow B_x] \)
(3) \( \exists r B_r \)
(4) Some ravens are black.
(5) \( \exists x [R_x \& B_x] \)
(6) \( \exists r B_r \)

The usual way in which the restriction is stated is set-theoretic, as in (7). Another way to state it is mereological, as in (8).

(7) For all \( r \) (or: There is an \( r \)) such that \( r \in R \), where \( R \) is the set of ravens
(8) For all \( r \) (or: There is an \( r \)) such that \( R \geq r \), where \( R \) is the totality of ravens

The latter formulation allows the variable to range not only over atomic (singular) individuals, but also over sums of individuals (plural individuals). This added power turns out to be useful, for example to distinguish between the meanings of (4) and (9). In (4) \( r \) ranges over all individual ravens, singular and plural, whereas in (9) it ranges over singular ones only, the difference being signaled by the morphology.

(9) Some raven is black.

This difference is sharpened when we consider (10) and (11), with the nondisjunctive predicate be noisy; not only are different numbers of ravens involved, but also (10) is ambiguous, depending on whether it is understood distributively or not. We may represent the logical forms of (10) and (11) as in (12) and (13). Note that (12)b entails (12)a but not conversely.

(10) Some ravens are noisy.
(11) Some raven is noisy.

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3 The formula \( R \geq r \) is interpreted “\( r \) is part of \( R \”).
4 The quantifier is different depending on whether the individual is singular or plural; see the analysis of example (10).
5 The variable \( r_1 \) ranges over atomic individual ravens; \( r_{11} \) ranges over nonatomic sums of ravens (\( \Pi \) for “plural”), and \( r \) (without a subscript) ranges over all individual (atomic and nonatomic) ravens. The quantifier \( \Pi x \) is understood as “there is more than one \( x \) such that ...”; it is equivalent to the quantifier \( 2x \) in the Quantification handout. The property \( N \) is be noisy.
(12) Logical forms for (10):
   a. $\exists r_1 \forall N R$
   b. $\forall r_1 \forall N R$

(13) Logical form for (11): $\forall r_1 \forall N R$

The universal quantifier counterpart to the existential quantifier in (11) is expressed by (14) and (15); its logical form is given in (16).\(^6\) There is no universal quantifier counterpart to the existential quantifier in (10).

(14) All ravens are noisy.
(15) Every/Each raven is noisy.
(16) Logical form for (14) and (15): $\exists r_1 \forall N R$

**Partitives**

Closely related to (10), (11), (14) and (15) are the partitive sentences (17) through (20), the difference being that the totality $R$ in (10), (11), (14) and (15) is the sum of all ravens, whereas in (17) through (20) it is a contextually delimited totality.

(17) Some of the ravens are noisy.
(18) One of the ravens is noisy.
(19) All (of) the ravens are noisy.
(20) Every one /Each (one) of the ravens is noisy.

**Restricted quantification over mass terms**

If the mass noun *oil* replaces the count noun *ravens* in (1) and (4), as in (21) and (22), the distinction between singular and plural is neutralized, and the quantifier ranges freely over parts of the totality of *oil* as in (23) and (24), without commitment to their being atomic parts. The same observation holds for the partitive sentences (25) and (26).

(21) All oil is black.
(22) Some oil is black.
(23) $A o B o$
(24) $E o B o$
(25) All of the oil is black.
(26) Some of the oil is black.

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\(^6\) In some contexts the phrase *all ravens* refers to a totality that consists entirely of ravens rather than to the totality of ravens itself; an example is *All ravens are in this cage.*
Relative size quantification

**Over count terms**

In its nonpartitive use, the quantifier *many* \((Mx)\) has two interpretations. In one it ranges over a plural individual indicating that it has a relatively large number of singular individuals as parts, and in the other it ranges over singular individuals indicating that the number of such individuals is relatively large. For example, the sentence (27) has the logical forms in (28). On the other hand, in its partitive use as in (29), *many* ranges only over singular individuals with an interpretation corresponding to (28)b.

(27) Many ravens are noisy.

(28) Logical forms for (27):
   a.  \(Mr_\Pi NR\)
   b.  \(Mr_1 NR\)

(29) Many of the ravens are noisy.

The number of individuals that constitutes the threshold of *many* is larger than the totality when the latter is small, so that a sentence like (30) is contradictory.

(30) Many of these three ravens are noisy.

The threshold also grows with the size of the totality, but obviously less slowly, since otherwise there would be no occasion to use it. As a result, a size for the totality is eventually reached at which a sentence like (31) is not a contradiction.

(31) Many of the ravens are noisy and many of the ravens are not noisy.

Similar remarks hold for the quantifier *a few*, which, when it ranges over plural individuals, indicates that it has a relatively small number of singular individuals as parts, and when it ranges over singular individuals indicates that the number of such individuals is relatively small.

**Over mass terms**

The quantifier *much* ranges over the individual parts of a mass totality indicating that the part is large, and in the case of its partitive use its threshold is always smaller than the totality. Consequently (32) is true only if the amount of oil that is black is large. On the other hand, (33) is true if the amount of oil that is black is a large part of the totality of the oil, no matter how large or small that amount is.

(32) Much oil is black.

(33) Much of the oil is black.

Similar remarks hold for the quantifier *a little*, except that the discussion concerning the threshold is irrelevant.
Proportional quantification

The quantifier *most* ranges over singular individuals comprising the totality denoted by a count noun to indicate that the number of such individuals is greater than half of the number of singular individuals that comprise the totality. It ranges over the parts of the totality denoted by a count noun to indicate that the amount of the part is greater than half of the amount of the totality. Unlike other proportional quantifiers (such as *half*), *most* has both nonpartitive and partitive uses, as in the following examples.

(34) Most ravens are noisy.

(35) Most of the ravens are noisy.

(36) Most oil is black.

(37) Most of the oil is black.

Generic quantification

The interpretation of (38) differs from that of (1) inasmuch as the former allows “exceptions”, i.e. it is not falsified by the existence of a tiny number of ravens that are not black; similar remarks hold for (39) in comparison to (21). These observations suggest that (38) and (39) involve generic quantifiers with interpretations like those in (40) and (41).

(38) Ravens are black.

(39) Oil is black.

(40) For all but a negligible number of atomic \( r_1 \) such that \( R \geq r_1 \)

(41) For all but a negligible number of tiny parts \( o \) such that \( O \geq o \)

Assuming that the threshold for constituting a negligible number of atomic or tiny parts grows with the totality of the domain, these generic quantifiers are necessity operators just like the universal quantifiers, but are weaker than them.

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