LING 501, Fall 2004: Mereology

Mereological implication structures

In the handout on the semantics of connectives, I introduced the notion of a mereological (part-whole) implication structure (MIS), and illustrated it with the Hasse diagram in Figure 1. In this structure, the atomic individuals a, b and c are the singular individuals, and the composite individuals a+b, a+c, b+c, and a+b+c are the plural individuals.

![Figure 1 A simple MIS](image)

**Predications over plural individuals**

The formula Pa+b represents a one-place predication on the plural individual a+b. Whether Pa+b is equivalent to Pa ∧ Pb depends on P. For example, if P is *be light (in weight)*, then Pa+b entails Pa ∧ Pb, but not conversely. On the other hand, if P is *be heavy*, then Pa & Pb entails Pa+b, but not conversely. Finally, if P is *be friendly*, then Pa+b is equivalent to Pa & Pb. If Pa+b entails Pa & Pb, then P is dissective. If Pa & Pb entails Pa+b, then P is cumulative. Thus, *be light* and *be friendly* are dissective, and *be heavy* and *be friendly* are cumulative. If Pa+b is equivalent to Pa & Pb, then P is cumulodissective.

The English sentence *Alice and Bob are heavy* is ambiguous; its meaning can be represented as either Ha+b or as Ha & Hb, suggesting that plural phrases like *Alice and Bob* have two analyses semantically, a collective analysis as the mereological sum of Alice and Bob, and a distributive analysis as the set {Alice, Bob}, with the understanding that P{a, b} is equivalent to Pa & Pb for all P. Consequently *Alice and Bob are friendly* also has the analyses Ma+b and M{a, b}, these being logically equivalent since *be friendly* is cumulodissective.

**Indefinite individuals**

MISs can be extended to include indefinite individuals, for example the individual 1, which is any of the singular individuals in the structure (i.e. a!b!c, where '!' indicates indefinite mereological disjunction); 2, which is any of the dual individuals (i.e. [a+b]![a+c]![b+c]); etc. The extension of Figure 1 is shown in Figure 2.

The introduction of indefinite individuals means that disjunctions of singular individuals can be formed; in particular 1 is the disjunction of a, b and c in the MIS under consideration. However, overlapping plural individuals no longer have disjunctions, unless additional partially indefinite individuals are introduced, such as a+[b!c]. Then the disjunction of a+b with a+c is a+[b!c].
Any formula of the form $P_1$ is equivalent to $\exists x \, P_x$, but $P_n$ is equivalent to $n \, x \, P_x$ and $P_\perp$ is equivalent to $A x \, P_x$ if and only if $P$ is cumulodissective. If $P$ is dissective but not cumulative, then $P_n$ entails $n \, x \, P_x$, and $P_\perp$ entails $A x \, P_x$, but not conversely. If $P$ is cumulative but not dissective, then $n \, x \, P_x$ entails $P_n$, and $P_x \, A x \, P_x$ entails $P_\perp$, but not conversely.

**Variables ranging over pairs, etc.**

Although $P_2$ is not equivalent to $2 \, x \, P_x$ except when $P$ is cumulodissective, $P_2$ is equivalent to $\exists x^2 \, P_{x^2}$, where $x^2$ is a variable which ranges over pairs of singular individuals. As a consequence, the collective and distributive interpretations of *two students are noisy* can be represented as $\exists x^2 \, [S_{x^2} \land N_{x^2}] \leftrightarrow \exists s^2 \, N s^2 \leftrightarrow N_2 s$ ‘a pair of students is noisy’ and $2 \, x \, [S_x \land N_x] \leftrightarrow 2s \, N_s$ ‘two students are each noisy’ respectively, where $2s$ represents an indefinite pair of students. The sentence *two pairs of students are noisy* should also have collective and distributive interpretations, namely $\exists s^2 \, N s^2 = N_2(2s)$ ‘a pair of pairs of students is noisy’ and $2s^2 \, N s^2$ ‘two pairs of students are each noisy’ respectively.

The distributive interpretation of *the students are noisy* is represented as $A x \, [S_x \rightarrow N_x] \leftrightarrow A s \, N_s$. The collective interpretation is represented as $E x^\perp \, [S_{x^\perp} \land N_{x^\perp}] \leftrightarrow E s^\perp \, N_{s^\perp} \leftrightarrow N_\perp s$.

**Mereological structures for mass individuals**

Mass individuals differ from count individuals in that atomic mass individuals do not have to be assumed; consequently sums of atomic mass individuals are also not assumed. Letting $\perp$ represent the totality of some unspecified mass individual, and assuming that it is a large totality, then $\mu$ represents an indefinite large part of that individual, $\sigma$ an indefinite part not specified for amount, and $\pi$ an indefinite small part. These individuals are related mereologically in Figure 3.

![Figure 3. MIS for an unspecified mass individual](image)
Letting the totality be water, then the statement *there is a little water* may be represented as $W\pi x = \text{Ex}\pi Wx\pi$, *there is water* and *there is some water* as $W\sigma x = \text{Ex}\sigma Wx\sigma$, *there is much water* as $W\mu x = \text{Ex}\mu Wx\mu$, and *everything is water* as $W\perp = \text{Ax}\perp Wx\perp$.

The paucal and multal variables $x\pi$ and $x\mu$ range over mass individuals only and can only be quantified existentially, whereas the paucal and multal quantifiers range over singular individuals only.

Because mass individuals are not interpreted as sums of atomic individuals, they are understood collectively but not distributively. For example, the sentence *the water is heavy* is analyzed only as $\text{Ex}\perp [Wx\perp & Hx\perp] = \text{H}\perp w$. Since the predicate *be heavy* is not dissective, one cannot conclude that any proper part of the water is heavy. Similarly the sentence *the water is cold* is analyzed only as $\text{Ex}\perp [Wx\perp & Cx\perp] = \text{C}\perp w$. However, since *be cold* is dissective, then one can conclude that every identifiable part of the water is cold; that is $\text{Ax}\pi [Wx\pi \rightarrow Cx\pi] = \text{Aw}\pi Cw\pi$. 