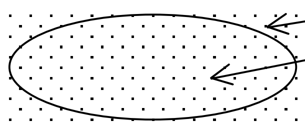
564 Lecture 24 Nov. 18, 1999

1 Note: Venn diagrams of intransitive Generalized Quantifiers

Cast your minds back to two lectures ago, when I introduced GQs in about three seconds before Howard Lasnik's talk. I had drawn some Venn diagrams to illustrate the notion of relations between sets and sets of sets, showing that this relation was the same as the relation between individuals and sets. I drew the pictures below:

1. Set-theoretic picture of intransitive predicates, e.g. *Ann arrived, is gray, is a cat*:

(a) If *Ann is gray* is true, then this is a good representation of the world:



{x: x is gray}

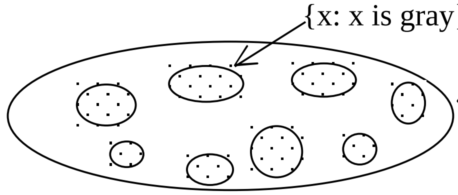
Ann

(b) Our semantics says “Ann is gray” is true iff Ann {y: y is gray}: it says that the meaning of **gray** is a function (relation) between individuals and truth values, conditioned by the grayness of the individual

3. Set-theoretic picture of intransitive quantifiers, e.g. *Everything arrived, is gray, is a cat:*

(a) [[**everything**]] = [λf $D_{<e,t>}$ **.** for all x D_e , f(x) =1]

{x: x is gray}

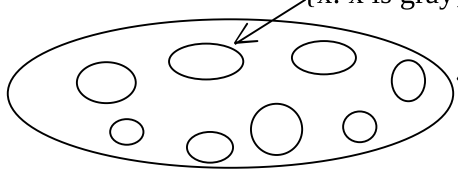


{X : D X} (or, in words: the set of sets X such that D is a subset of X.)

(b) [[**nothing**]] = [λf $D_{<e,t>}$ **.** for no x D_e , f(x) =1]

“Nothing is gray”

{x: x is gray}

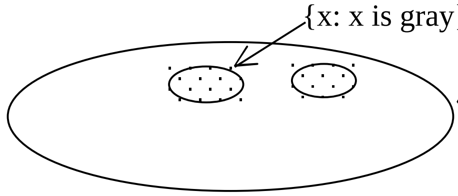


{X : D ∩ X = } (in words: the set of sets X such that no member of D is in X)

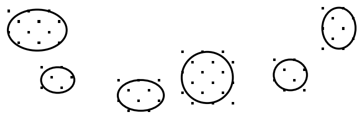
(c) [[**something**]] = [λf $D_{<e,t>}$ **.** for some x D_e , f(x) =1]

“Something is gray”

{x: x is gray}



{X : D ∩ X ≠ } (in words: the set of sets X such

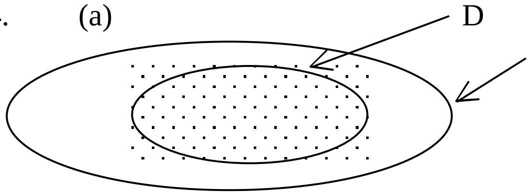


that at least one member D is in X.)

And the pictures in (a) and (b) in particular caused me some worry. A set is identified by its members, and any particular thing can only be a member of a set once. So my pictures in (a) and (b) have to be wrong. I've drawn them as if there were several instances of D contained within the set denoted by "everything" and several instances of in the set denoted by "nothing". And I worried that this was a problem.

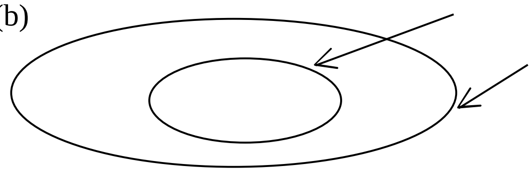
But, in fact, it's not. It's true that I should have drawn the set denoted by "Everything" and "Nothing" as follows:

4. (a)



{X : D X} (or, in words: the set of sets X such that D is a subset of X.)

(b)

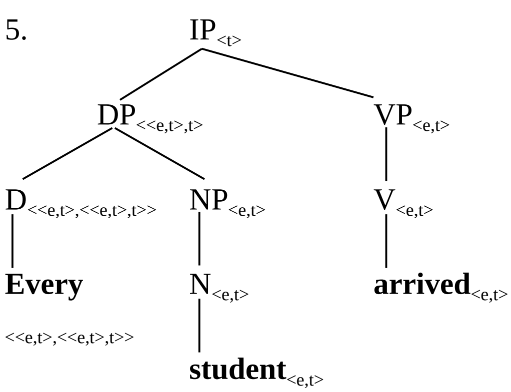


{X : D ∩ X = } (in words: the set of sets X such that no member of D is in X)

And what "Everything" does, when it's predicated of a set, is identify that set with D, and "Nothing", when predicated of a set, identifies that set with . So the Venn diagrams aren't as illustrative as I'd hoped, but you still get the idea, yes? GQs are functions that express relations between sets.

2 Quick review: the definitions of 2-place quantifiers

5.



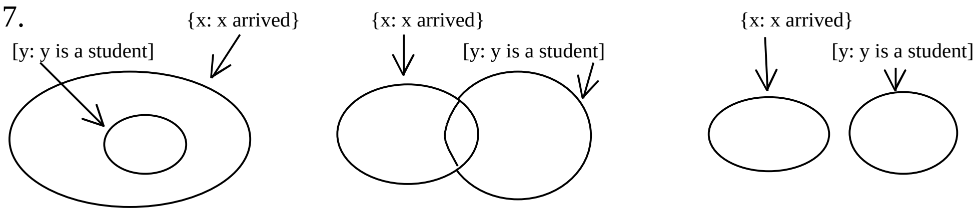
Here, we characterize the meaning of **every** as being conditioned, not by the relationship of the VP set to D_e, but by the relationship of the VP set to the set described by the NP. Here's the denotation of the three determiners we've got under consideration:

6. $[[\textbf{every}]] = [\lambda f \quad D_{<e,t>} \,.\, [\lambda g \quad D_{<e,t>} \,.\, \{x: f(x)=1\} \quad \{y: g(y)=1\}]]$

   $[[\textbf{some}]] = [\lambda f \quad D_{<e,t>} \,.\, [\lambda g \quad D_{<e,t>} \,.\, \{x: f(x)=1\} \cap \{y: g(y)=1\} \neq \quad]]$

   $[[\textbf{no}]] = [\lambda f \quad D_{<e,t>} \,.\, [\lambda g \quad D_{<e,t>} \,.\, \{x: f(x)=1\} \cap \{y: g(y)=1\} = \quad]]$

7.



every **some** **no**

Given the definitions in 6, let's just quickly work out the truth-conditions of the sentence "No babies are illogical". Notice especially what happens at the VP node (we simply ignore the presence of "is") and how the functions are plugged into the defintion of **no** in turn. The function that is the immediate argument of "no" (i.e. in this case, "babies") is called the *restrictor*.

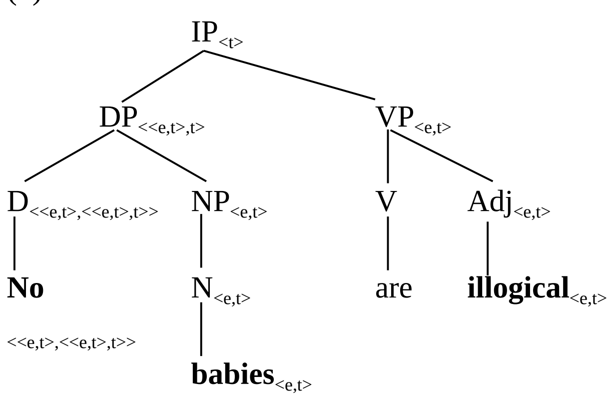
8. (a) Lexical items

   $[[\textbf{illogical}]] = [\lambda x \quad D_e \,.\, \text{x is illogical}]$

   $[[\textbf{babies}]] = [\lambda x \quad D_e \,.\, \text{x is a baby}]$

   $[[\textbf{no}]] = [\lambda f \quad D_{<e,t>} \,.\, [\lambda g \quad D_{<e,t>} \,.\, \{x: f(x)=1\} \cap \{y: g(y)=1\} = \quad]]$

(b) The tree



(c) Putting the elements of the tree together to find out what the tree means:

$[[\text{IP}]] = [[\text{DP}]]([[\text{VP}]])$ by FA

= [[DP]]([[**illogical**]]) by NN 2x

= [[D]]([[NP]])([[**illogical**]]) by FA

= [[**No**]]([[**babies**]])([[**illogical**]]) by N.N. 3x

= [[**No**]]([λx D_e **.** x is a baby])([λy D_e **.** y is illogical]) by L.T.

= [λf $D_{<e,t>}$ **.** [λg $D_{<e,t>}$ **.** {x: f(x)=1} ∩ {y: g(y)=1} =]]([λx D_e **.** x is a baby])([λy D_e **.** y is illogical])

by L.T.

= [λg $D_{<e,t>}$ **.** {x: [λx D_e **.** x is a baby](x)=1} ∩ {y: g(y)=1} =]([λy D_e **.** y is illogical])

by def. of λ applied to λf

= 1 iff {x: [λx D_e **.** x is a baby](x)=1} ∩ {y: [λy D_e **.** y is illogical](y)=1} =

by def. of λ applied to λg

= 1 iff {x: x is a baby} ∩ {y: y is illogical} =

by def of λ applied to λx and λy

i.e. = 1 iff no baby is illogical.

3 Presuppositionality and GQ

Remember that we defined "the" as carrying a presupposition that there's only one instantiation of its argument predicate in the domain of discourse? The way we did this is we limited its domain: "the" is only allowed to take as arguments functions which are true of only one thing in the domain of discourse. Here's our definition of "the":

9. [[**the**]] = [λf $D_{<e,t>}$ such that there is only one y for which f(y) =1 **.** the individual x such that f(x) =1]

Remember that this meant that if we said "the dog" in a situation where there's more than one salient dog, or no salient dog, that our sentence wouldn't get a truth value -- it wouldn't be true or false, just undefined.

(Note that the salient difference between the present analysis of **the** and the famous Russellian one where **the** is a quantifier is that "the dog is black" is false for Russell in a situation where there's more than one dog, but it's simply undefined for us.

The question of this section is, do other determiners, in particular, quantifiers, carry similar restrictions?

The answer appears to be yes. There are a couple of quantifiers for whom a non-presuppositional analysis doesn't seem to work at all.

10. **Both** and **neither**: are these sentences true or false when there isn't two cats?

(a) Both cats have stripes.

(b) Neither cat has stripes.

If we're looking at a bunch of cats, more or less than two, and they're all striped, then is (a) or (b) false or t rue? H&K claim, and I agree, that the judgement is that in fact they are simply undefined, in the same way that "The cat is striped" would be in that situation. So given this intuition, we can state that like "the", the felicitous use "both" and "neither" involves a certain presupposition: the restrictor set must have only two members (in the domain of discourse). They must be quantifiers that have a restriction on the possible domains they can apply to. Their definitions will look like this:

11.

(a)

$[[\textbf{both}]] = [\lambda f \quad D_{<e,t>}$ such that $|\{x: f(x)=1\}|=2 \,.\, [\lambda g \quad D_{<e,t>} \,.\, \{x: f(x)=1\} \quad \{y: f(y)=1\}]]$

(b)

$[[\textbf{neither}]] = [\lambda f \quad D_{<e,t>}$ such that $|\{x: f(x)=1\}|=2 \,.\, [\lambda g \quad D_{<e,t>} \,.\{x: f(x)=1\} \cap \{y: f(y)=1\} = \quad]]$

Now, to understand what these say, let's compare them to the definitions for **every** and **no** that we gave above. We said that "Every cat has stripes" is true just in case the set of cats is a subset of the set of things that have. Now here, our truth-condition is exactly the same: "Both cats have stripes" is true just in case the set of cats is a subset of the set of things that have stripes. *But*, there's an additional rider: the use of **both** is only appropriate when the set of cats only has two members. Similarly for the relationship between **neither** and **no**: **neither** has exactly the same truth conditions as **no**, but it's only appropriately used when the restrictor set only has two members. (This is the answer to the exercise on p. 154)

Why are we putting the condition on the number of elements in the set in the domain condition, rather than in the truth conditions? Consider what would happen if we put it in with the truth conditions:

12. *A non-presuppositional definition for **both***

$[[\textbf{both}]] = [\lambda f \quad D_{<e,t>} \,.\, [\lambda g \quad D_{<e,t>} \,.\, \{x: f(x)=1\} \quad \{y: f(y)=1\}$ and $|\{x: f(x)=1\}| =2]]$

What happens is that we never get the value "undefined" for a sentence like "Both cats are striped" when there's more or fewer than two cats in the world; what we get instead is the truth value "false". So in the situation described in (10) above, the sentences would be false if there's three cats and they're all striped, or if there's three cats and none of them are striped. This doesn't seem to accord with speakers' judgements. Worse yet, say H&K, what if we add a propositional connective like "not" to a sentence that the non-presuppositional analysis predicts to be true? Then we get

13. (a) H&K's ex. of a wrong prediction of the non-presupp. definition
[[**I didn't see both cats**]] = 1 if there's one cat and I saw it.
(b) A slightly more fair example sentence:
[[**It is not the case that both cats are striped**]] = 1 if there's three cats and they're all striped.

The non-presuppositional definition, too, will turn out the truth value True in these situations, but (claim H&K) it's not "true", it's simply undefined.

At this point it might be worth considering briefly the difference between truth and appropriateness. Presumably a proponent of the non-presuppositional analysis might say, it *is* true, but simply not appropriate in a situation like that. It's like saying, "The sky is blue" in response to the question, "Who went to the store?" -- the answer is true, but not appropriate. In my judgement, though, the two situations are different, in that we can still clearly judge "the sky is blue" to be true in the above situation, but it's still difficult to decide whether "It's not the case that both cats are striped" is true or not. (How would a literal-minded child respond to these utterances in the described situation?)

Now, the presuppositional analysis of **both** and **neither** has consequences for our choice of definitions for all those properties we described above, because the presupposition means that they are no longer total functions from $D_{<e,t>}$, but only partial ones (just as was the case for **the**) -- but given our time constraints, I'm not going to go into it in depth. The exercises on p. 157-159 seems to imply that this has consequences for the analysis of "there" insertion. I'll look at this over the weekend and if it's interesting we may return to it on Tuesday.

Note that at this point, we could treat "the" as a quantifier too, exactly like "both" except that the cardinality of the restrictor would be presupposed to be 1. The relationship between this type of analysis and the Russellian one would be the same (I think!) as the relationship between the presuppositional analysis of **both** we're assuming and the non-presuppositional analysis of **both** that we ridiculed in 12 and 13 above.

4 Are all quantifiers presuppositional?

Some standard inferences of Aristotelian logic seem to imply that, in fact, all quantifiers must be presuppositional, and that we'll have to revise all the definitions which we've derived above to include the presupposition that the set of the restrictor is not empty. (Confusingly, the definitions we've arrived at, the "modern" ones, are sometimes called the "classical" definitions -- this is "classical" within the framework of post-Fregean logic, not "classical" in the sense of "in Greek antiquity", because if Aristotle had had access to generalized quantifier theory, he no doubt would have advocated presuppositional definitions rather than modern definitions, because of these facts we're about to see, if I ever finish this sentence.)

14. A Contradiction, A Tautology, and Two Inferences from Aristotelian Logic:
   (a) "Every A is B" and "No A is B" is a contradiction.
   (b) "Some A are B or some A are not B" is a tautology.
   (c) "Every A is B" entails that "Some A are B".
   (d) "No A are B" entails "Some A are not B".

You'll remember these from our initial investigation of first-order logic. They correspond to English sentences of the following form:

15. (a) Every woman is a smoker and no woman is a smoker.
   (b) Some women are smokers or some women are not smokers.
   (c) Every woman is smoker, therefore some women are smokers.
   (d) No woman is a smoker, therefore, some women are not smokers.

Now, the definitions for quantifiers that we've given mean that the Aristotelian view of their necessary truth or falsity no longer holds. In fact, because the empty set is a member of all sets, according to our definitions the truth of these statements (a is a contradiction, b is a tautology and c and d are valid inferences) no longer holds, in the particular cases when the restrictor is empty.

The claim is that, in order for the Aristotelian inferences to hold, all these quantifiers carry a presupposition that their restrictor set is non-empty. So, their definitions will look like this:

16. [[**every**]] = [$\lambda f \quad D_{<e,t>}$ s.t. $\{x: f(x)=1\} \neq \quad$. [$\lambda g \quad D_{<e,t>}$. $\{x: f(x)=1\} \quad \{y: g(y)=1\}$]]
   [[**some**]] = [$\lambda f \quad D_{<e,t>}$ s.t. $\{x: f(x)=1\} \neq \quad$. [$\lambda g \quad D_{<e,t>}$. $\{x: f(x)=1\} \cap \{y: g(y)=1\} \neq \quad$]]
   [[**no**]] = [$\lambda f \quad D_{<e,t>}$ s.t. $\{x: f(x)=1\} \neq \quad$. [$\lambda g \quad D_{<e,t>}$. $\{x: f(x)=1\} \cap \{y: g(y)=1\} = \quad$]]

So, how can we decide between these two approaches? The first piece of evidence in favor of the presuppositional analysis to look for is speaker's judgements about the truth or falsity of sentences like those in 15. What do you think?

Homework: