## 1 Homework

In our current universe, " $x$ " corresponds fairly closely with what we used " $c$ " to do in Universal Instantiation - be an arbitrary individual. We used "c" to stand for an arbitrary individual that we could use to make the laws of inference applicable to the predicate calculus structure. Because c was arbitrary, we could then use Universal Generalization to get back to a structure with the quantifier in it.

Here, in a similar way, we've used the value " $x$ " to create a structure in which the trace has the value of an arbitrary individual. We then can apply Predicate Abstraction one of our rules of interpretation -- to the node whose interpretation is dependent upon the assignment " x " and produce a meaning for that node: it's a function whose domain ranges over possible values for $x$. The interpretation of the function is independent of any assignment. We can only do this because " $x$ " stands for an arbitrary individual, not some specific one.

So, one way in which the function of x is similar to the function of c is that they allow us to apply some rule which normally can't be applied when a variable gets a specific interpretation under some assignment.

This begins to touch on the important question of what exactly is variable binding, which H\&K discuss in section 5.4. I won't go into it, except very briefly right now; for more information, look at their section.
(a) a variable is "variable" because its interpretation may vary from assignment to assignment. Things that aren't variables have the same interpretation under any assignment.
(b) Binding removes the assignment-dependence of a variable. In "John loves himself", we can say that "John" binds "himself" because the only interpretation of "himself" is one in which "himself" = "John" - by the time you have interpreted the whole sentence, the meaning of the sentence is assignment-independent. In contrast, in "John loves him", "him" is free because the interpretation of "him" is necessarily dependent on the assignment function in use at the moment of utterance. This is a slightly tricky notion, because the assignment function-dependence of a variable can sometimes make it seem like "free" variables are in fact less "free" than "bound" variables - they're chained to the interpretation of the assignment function. That's not the case in "John likes himself", because the meaning of "himself" is so determinedly locked in - but consider the value of the trace in the relative clause " $\mathrm{who}_{1}$ John loves $\mathrm{t}_{1}$ ". Here, the trace is bound, not free. Why is it bound? because the meaning of the whole clause is assignmentindependent: it's $\left[\lambda \mathrm{x} \in \mathrm{D}_{\mathrm{e}}\right.$. John loves x$]$ - this is its meaning under any assignment This is due to the trick with the relative pronoun where we made it modify whatever
assignment we' d been considering to assign the value x to the index of its trace, and then pick out the set of $x$ 'es that give the value True when the $t=x$. That set will be the same no matter what assignment function we're considering, hence, it's assignmentindependent and the variable represented by t is bound - even though the result of its binding is a function where it seems like you can consider a much larger range of possible values for $t$ and hence it might seem "freeer" than it does under an assignment, when its meaning has to be whatever the assignment function says. But really, the value of $t$ is in fact "bound", and it can seem intuitively so when you consider the fact that "who John loves" will always denote the set of individuals with that one darn property of being loved by John, no matter what — but "him" just by itself can denote any individual in the universe, with any property at all, if the assignment function says it does.

Ok, that's it for bound and free. Read H\&K for deeper insight.
2. (a) $a^{\text {Betty/3 }}=\quad 2 \rightarrow$ Betty
$3 \rightarrow$ Betty
$4 \rightarrow$ Fred
$6 \rightarrow$ Wilma
$8 \rightarrow$ Barney
(b) $\quad \mathrm{a}^{\text {Dino/1, Barney/2 }}=1 \rightarrow$ Dino
$2 \rightarrow$ Barney
$4 \rightarrow$ Fred
$6 \rightarrow$ Wilma
$8 \rightarrow$ Barney
(c) $\quad \mathrm{a}^{\text {BamBam/4, Pebbles } 6, \text { Betty/8 }}=2 \rightarrow$ Betty
$4 \rightarrow$ BamBam
$6 \rightarrow$ Pebbles
$8 \rightarrow$ Betty
(d) $\quad \mathrm{a}^{\text {BamBam/1, Fred/2, Wilma/3, Barney/4, Pebbles/5, Dino/7 }}=1 \rightarrow$ BamBam
$2 \rightarrow$ Fred
$3 \rightarrow$ Wilma
$4 \rightarrow$ Barney
$5 \rightarrow$ Pebbles
$6 \rightarrow$ Wilma
$7 \rightarrow$ Dino
$8 \rightarrow$ Barney



licked
Fred
$[[i s]]=\left[\lambda x \in D_{e}\left[\lambda y \in D_{e} \cdot y=x\right]\right]$
a. $\quad[[\mathrm{IP}]]=[[\mathrm{VP}]]([[\mathrm{DP}]])$
by F.A.
b. $=[[\mathrm{VP}]]([[$ Dino $]])$
by N.N.
c. $=[[\mathrm{VP}]]($ Dino $)$
by L.T.
d. $=[[\mathrm{V}]]([[\mathrm{DP}]])($ Dino $)$
by F.A.
e. $=[[i s]]([[D P]])($ Dino $)$
by N.N.
f. $\quad=[[$ is $]]([[D]]([[N P]]))($ Dino $)$
by F.A.
g. $=[[$ is $]]([[$ the $]]([[N P]]))($ Dino $)$ by N.N.
h. $\quad=[[$ is $]]\left([[\right.$ the $]]\left(\left[\lambda x \in\right.\right.$ D. $\left.\left.\left.[[\operatorname{AdjP]}]](x)=\left[\left[N^{\prime}\right]\right](x)=1\right]\right)\right)($ Dino $)$ by P.M.
i. $\quad=[[$ is $]]\left([[\right.$ the $]]([\lambda x \in$ D . [[brown $\left.\left.\left.]](x)=\left[\left[N^{\prime}\right]\right](x)=1\right]\right)\right)($ Dino $) \quad$ by N.N. $2 x$
j. $\quad=[[$ is $]]([[$ the $]]([\lambda x \in \operatorname{D} .[[b r o w n]](x)=[\lambda y \in D .[[N]](y)=[[C P]](y)=1](x)$
$=1])$ )(Dino)
by P.M.
k. $\quad=[[$ is $]]([[$ the $]]([\lambda x \in$ D . $[[$ brown $]](x)=[\lambda y \in D$. $[[$ dinosaur $]](\mathrm{y})=[[\mathrm{CP}]](\mathrm{y})=1](\mathrm{x})=1]))($ Dino $)$ by N.N.

1. (ok: subpart of the proof: I'm going to derive the meaning for CP without all the other information, then substitute it back into $k$ in the appropriate place)
i. $[[C P]]=\left[\lambda z \in D .\left[\left[C^{\prime}\right]\right]^{z}=1\right] \quad$ by P.A.
ii. $=\left[\lambda z \in \mathrm{D} \cdot[[\mathrm{IP}]]^{z}=1\right] \quad$ by N.N.
iii. $\quad=\left[\lambda z \in \mathrm{D} \cdot[[\mathrm{VP}]]^{2}\left([[\mathrm{DP}]]^{2}\right)=1\right] \quad$ by F.A.
iv. $\quad=\left[\lambda z \in \mathrm{D} \cdot[[\mathrm{VP}]]^{2}\left(\left[\left[\mathrm{t}_{1}\right]\right]^{2}\right)=1\right] \quad$ by N.N.
v. $=\left[\lambda z \in \mathrm{D} \cdot[[\mathrm{VP}]]^{2}(\mathrm{z})=1\right] \quad$ by P.T.
vi. $\quad=\left[\lambda z \in D \cdot[[\mathrm{~V}]]^{z}\left([[\mathrm{DP}]]^{z}\right)(\mathrm{z})=1\right] \quad$ by F.A.
vii. $=\left[\lambda z \in \mathrm{D} \cdot[[\mathrm{V}]]^{2}\left([[\text { Fred }]]^{2}\right)(\mathrm{z})=1\right] \quad$ by N.N.
viii. $=\left[\lambda z \in D \cdot[[V]]^{z}(\right.$ Fred $\left.)(z)=1\right] \quad$ by L.T.
ix. $\quad=\left[\lambda z \in D .[[\text { licked }]]^{z}(\right.$ Fred $\left.)(z)=1\right] \quad$ by L.T.
$x . \quad=[\lambda z \in D .[\lambda x \in D .[\lambda y \in D . y$ licked $x]]($ Fred $)(z)=1] \quad$ by L.T.
xi. $=[\lambda z \in D .[\lambda y \in D . y$ licked Fred $](z)=1] \quad$ by def of lambda
xii. $=[\lambda z \in D . z$ licked Fred $] \quad$ by def of lambda
ok: back to bigger picture:
2. $=[[$ is $]]([[$ the $]]([\lambda x \in$ D . $[[$ brown $]](x)=[\lambda y \in D$.
$[[$ dinosaur $]](y)=[\lambda z \in D . z$ licked Fred $](y)=1](x)=1]))($ Dino $)$ by subproof above
m. $=[[$ is $]]([[$ the $]]([\lambda x \in \mathrm{D} .[[$ brown $]](x)=[\lambda y \in \mathrm{D}$.
$[\lambda w \in D . w$ is a dinosaur $](y)=[\lambda z \in D . z$ licked Fred $](y)=1](x)=1])($ Dino $)$
by L.T.
n. $\quad=[[\mathbf{i s}]]([[$ the $]]([\lambda x \in \mathrm{D} .[[$ brown $]](\mathrm{x})=[\lambda \mathrm{y} \in \mathrm{D} \cdot \mathrm{y}$ is a dinosaur and y licked Fred] $(x)=1])$ )(Dino)
by def. of lambda
o. $\quad[[$ is $]]([[$ the $]]([\lambda x \in D .[\lambda z \in \mathrm{D} . \mathrm{z}$ is brown $](x)=[\lambda y \in \mathrm{D} \cdot \mathrm{y}$ is a dinosaur and y licked Fred] $(x)=1])$ )(Dino)
by L.T.
p. $\quad[[i s]]([[$ the $]]([\lambda x \in D . x$ is brown and $x$ is a dinosaur and $x$ licked Fred $]))($ Dino $)$ by def. of lambda
q. $\quad[[i s]]\left(\left[\lambda f \in D_{<e, t}\right.\right.$ and there is only one $x \in D$ such that $f(x)=1$. the individual $y \in$ $D$ such that $f(y)=1]([\lambda x \in D . x$ is brown and $x$ is a dinosaur and $x$ licked Fred]))(Dino)
by L.T.
r. $\quad[[i s]]($ the unique individual $\mathrm{y} \in \mathrm{D}$ such that $[\lambda \mathrm{x} \in \mathrm{D} . \mathrm{x}$ is brown and x is a dinosaur and x licked Fred] $(\mathrm{y})=1$ )(Dino)
by def. of lambda
s. $\quad[[\mathbf{i s}]]$ (the unique individual $\mathrm{y} \in \mathrm{D}$ such that y is a brown dinosaur who licked Fred)(Dino)
by def. of lambda
t. $\quad\left[\lambda x \in D_{e}\left[\lambda y \in D_{e} \cdot y=x\right]\right]$ (the unique brown dinosaur who licked Fred)(Dino)
by L.T. (and simplification of that big individual descr.
u. $\quad\left[\lambda y \in D_{e} \cdot y=\right.$ the unique brown dinosaur who licked Fred] (Dino) by def. of lambda
v. $\quad=1$ iff Dino $=$ the unique brown dinosaur who licked Fred.
phew!!

## 2 Quantified DPs don't denote individuals

So far, all DPs are of type <e>. We've seen both definite description DPs ("the brown dinosaur who licked Fred") and proper name DPs ("Fred", "Dino"). And these are the things that have served as arguments so far.

However, there's a whole class of things that can appear in argument positions (spec IP, complement to V, etc.) that we haven't treated so far, and which we will certainly need to arrive at a semantics for. They include DPs like the following:

1. (a) At most one person smokes.

Exactly three people smoke.
Few people smoke.
No people smoke.
Only John smokes.
(b) More than two people smoke

Some people smoke.
A person smokes.
Three people smoke.
(c) Everyone smokes.

All people smoke.
Most people smoke.

We have to figure out what an appropriate type for these DPs is. The easiest answer would be that they are of type e, individual members of the set $\mathrm{D}_{\mathrm{e}}$; that way, we wouldn't have to do very much work to accommodate them within our system.

However, it's not possible to treat them as entities. Let's look at some of the empirical arguments.
2. Does "Only John" refer to an individual, say, John?
(a) if only John refers to the individual John, then it should be the case that
(b) $[[$ Only John left $]]=1$ if John left.
(c) However, although [[John left $]]=1$ in a scenario where John and Sam left, our intuition says that in that situation, $[[$ Only John left $]]=0$. So only John must not refer to the individual John per se.

## 3. Subset-superset inferences:

(a) If $\mathrm{x} \in$ set A , and set $\mathrm{A} \subseteq$ set B , then $\mathrm{x} \in \mathrm{B}$, by set theory.
(b) The set of entities that smoked yesterday morning are a subset of the entities that smoked yesterday, so if John smoked yesterday morning, then we can infer correctly that John smoked yesterday.
(c) If only John or at most one person denotes an entity, then it should obey the rule of set theory given in (a). But:
(d) Only John smoked yesterday morning does not mean that we can therefore infer that Only John smoked yesterday. Ditto for At most one person smoked yesterday morning: it does not entail that At most one person smoked yesterday.
(Try this out for the other determiners listed above in (1)).

## 4. The Law of Contradiction

(a) If $\mathrm{A} \cap \mathrm{B}=\varnothing$, then for any $\mathrm{x} \in \mathrm{D}_{\mathrm{e}}$, the statement " $\mathrm{x} \in \mathrm{A}$ and $\mathrm{x} \in \mathrm{B}$ " must be false -- it will necessarily be a contradiction.
(b) The set of entities who smoke and the set of entities who don't smoke are disjoint, so the sentence John smokes and John doesn't smoke is a contradiction.
(c) If some person denotes an entity, then it should obey the rule of set theory given in (a). But:
(d) Some person smokes and some person doesn't smoke isn't a contradiction. (Try it out for the other determiners...)

## 5. The Law of Excluded Middle

(a) If $\mathrm{A} \cup \mathrm{B}=\mathrm{D}$, then the statement $\mathrm{x} \in \mathrm{A}$ or $\mathrm{x} \in \mathrm{B}$ " must be true -- it will necessarily be a tautology.
(b) The set of entities who smoke and the set of entities who don't smoke put together equal D -- an entity must either smoke or not smoke (habitual quitters notwithstanding). So John smokes or John doesn't smoke must be true.
(c) If Everybody denotes an entity, then it should obey the rule of set theory given in (a). But:
(d) Everybody smokes or everybody doesn't smoke isn't a tautology, not even close. Ergo, Everybody must not denote an entity.

More reasons to think that these quantified DPs aren't the same as individuals:

Some types of syntactic manipulation, when applied to entities in sentences, don't affect their truth conditions. That is, they're true in exactly the same real world scenarios, and false in exactly the same real world scenarios. The manipulations are things that speakers do to convey non-truth-conditional implications, like changing the emphasis or focus of a sentence. Here are three examples:
6 (a) Passivization
Engelbert kicked Humperdinck.
Humperdinck was kicked by Engelbert.
(b) Topicalization.

Engelbert likes peas.
(Dialogue:
A: What shall we serve Julio and Engelbert for vegetables? I know Julio likes beans, peas, cauliflower and spinach.

B: Now, peas, Engelbert likes. I don't know about anything else.
(c) Such-that

Only a few people really appreciate Engelbert.
Engelbert is such that only a few people appreciate him.

However, if rather than individuals, we plug in some of our mystery expressions, we find that these manipulations can in fact change truth conditions.

## $7 \quad$ (a) Passivization

Almost everybody answered at least one question.
At least one question was answered by almost everybody.
(b) Topicalization

Almost everybody answered at least one question.
At least one question, almost everybody answered.
Both passivization and topicalization change the truth conditions in the same way here. The first sentence could be true in a situation where there's 5 students and 5 questions, and students one through four answered questions one through four
respectively (but didn't answer any other question), and student 5 didn't answer any question. H\&K point out, though, that the passivized or topicalized version is not true of this situation (because it implies there's one specific question that almost all the students answered).
(c) Such that.

Nobody likes more than one singer.
More than one singer is such that nobody likes them.
Nobody is such that they like more than one singer.
Again, the sentence where "More than one" is the subject can be true even if there's some people who like a lot of singers, but in the sentences where "Nobody" is the subject that's not so (at least definitely not in the third sentence; some people can get it for the first, apparently).

Finally, these DPs, as we've already observed, can be ambiguous in ways that DPs that denote individuals are not. Here's an example:
8. (a) John doesn't like Julio Iglesias.
(b) John doesn't like more than two singers.
(a) just means one thing. (b) can be true if John doesn't like five singers (he doesn't like a number of singers that is more than two) but does like 4 others. (b) can also be false in that situation: we could imagine that $b$ means that it is not the case that more than two singers are such that John likes them. The ambiguity was introduced when we replaced "Julio Iglesias", an individual, with "more than two singers", a DP which must mean something else.

## 2 Quantified DPs don't denote sets of individuals.

(Problem on page 138-139).

Now, it's pretty intuitively obvious that DP like "Everything" doesn't denote an individual, and we've proved that to be true, above. (We've also proved that "only John" doesn't denote an individual, even though that's not so intuitively obvious). A seemingly intuitively obvious thing for "Everything" to refer to, though, is perhaps a set -- in fact, the set of all things, D. And similarly, "nothing" should perhaps also refer to a set, the null set $\varnothing$. This, however, ain't so. To see why it ain't so, let's look at the problem set of H\&K on page 138:

In this problem set, we assume that all DPs denote sets (even proper names -- they denote singleton sets, rather than individuals), and that "everything" and "nothing" have as denotations the set D and the set $\varnothing$, respectively. And predicates are functions that take sets as arguments, and give the value true if the argument set is a subset of the predicate set. Here's the new lexical entries:
$[$ [Ann $]]=\{$ Ann $\}$
[ Jacob] $]=\{$ Jacob $\}$
[ [everything] $]=\mathrm{D}$
$[[$ nothing $]]=\varnothing$
[[vanished]] $=\lambda X \in \operatorname{Pow}(D) . X \subseteq\{y: y \in D: y$ vanished $\}$
$[[$ reappeared $]]=\lambda X \in \operatorname{Pow}(D) \cdot X \subseteq\{y: y \in D: y$ reappeared $\}$
[[vanished fast $]] \lambda X \in \operatorname{Pow}(D) . X \subseteq\{y: y \in D: y$ vanished fast $\}$
and they assume the following relationship:
$\{y: y \in D: y$ vanished fast $\} \subseteq\{y: y \in D: y$ vanished $\}$
(a) Assuming standard phrase structures and compositionality rules, what does this semantics predict are the truth conditions of the following sentences?
(i) Ann vanished.
(ii) Everything vanished.
(iii) Nothing vanished.
"Ann vanished" is true if the set $\{A n n\}$ is a subset of the set of things that vanished. "Everything vanished" is true if the set of everything is a subset of the things that vanished -- and that will only be true if everything vanished. Trickily, though, "Nothing vanished" is true if $\varnothing$ is a subset of the set of things that vanished -- and we know from set theory that $\varnothing$ is a subset of all sets. So "Nothing vanished" is predicted to be true no mater what -- if nothing vanished, if Ann vanished, and if everything vanished.

What kind of lexical entry could a DP like Exactly two things get if we assume that DPs denote sets? What predictions does such a lexical entry make for a sentence like Exactly two things vanished?
(b) Which inferences are predicted to be valid in this semantics? Which inferences are in fact true in everyday life?
(i) Ann vanished fast.

Ergo, Ann vanished.
(ii) Everything vanished fast.

Ergo, everything vanished.
(iii) Nothing vanished fast.

Ergo, nothing vanished.
(c) Given the above discussion, you should be able to see what is wrong with the following:
(a) A is better than $\mathrm{B}, \mathrm{C}$ is better than A , there fore C is better than B . But what about:
(a) Nothing is better than Nirvana.
(b) A ham sandwich is better than nothing.
(c) Therefore, a ham sandwich is better than Nirvana.
cf. also the other joke: The rich need it, the poor have it, ... ?

