Harmonic OT

A. Overview

(1) The plan:
   a. definitions;
   b. how HG works;
   c. learning;
   d. implementation in R;
   e. predictions;
   f. Lango.

B. Definitions

(2) Harmony function
Let $C = \{C_1, \ldots, C_n\}$ be a set of constraints, and let $W$ be a total function from $C$ into positive real numbers. Then the harmony of a candidate $A$ is given by:

$$H_{C,W}(A) = \sum_{i=1}^{n} W(C_i) \cdot C_i(A)$$

(3) Assumptions here:
$W(C_i)$ always positive
$C_i(A)$ always negative

(4) Tableaux
A tableau is a structure $(A_{In}, C)$ where $A_{In}$ is a (possibly infinite) set of candidates sharing the input $In$ and $C$ is a (finite) constraint set.

(5) Optimality
Let $T = (A_{In}, C)$ be a tableau, and let $W$ be a weighting function for $C$. A candidate $A = \langle In, Out \rangle \in A_{In}$ is optimal iff $H_{C,W}(A) > H_{C,W}(A')$ for every $A' \in (A_{In} - \{A\})$.

(6) Tableau set
A tableau set is a pair $(T, C)$ in which $T$ is a set of tableaux such that if $T = (A_{In}, C') \in T$ and $T' = (A'_{In}, C'') \in T$ and $T \neq T'$, then $In \neq In'$ and $C = C' = C''$. 1
C. How HG works

(7) 

<table>
<thead>
<tr>
<th>weight</th>
<th>2</th>
<th>1</th>
<th>( \mathcal{H} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>( C_1 )</td>
<td>( C_2 )</td>
<td></td>
</tr>
<tr>
<td>a. Output(_a)</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>b. Output(_b)</td>
<td>-1</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>

(8) An impossible language?

/\text{ban}/ \rightarrow [\text{ban}]
/\text{bantan}/ \rightarrow [\text{banta}]/[\text{batan}]

(9) 

a. 

<table>
<thead>
<tr>
<th>weight</th>
<th>2</th>
<th>1</th>
<th>( \mathcal{H} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>/\text{bantan}/</td>
<td>NoCoda</td>
<td>Max</td>
<td></td>
</tr>
<tr>
<td>i. ban.tan</td>
<td>-2</td>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>ii. ba.tan</td>
<td>-1</td>
<td>-1</td>
<td>-3</td>
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</tbody>
</table>

b. 

<table>
<thead>
<tr>
<th>weight</th>
<th>1</th>
<th>2</th>
<th>( \mathcal{H} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>/\text{ban}/</td>
<td>NoCoda</td>
<td>Max</td>
<td></td>
</tr>
<tr>
<td>i. ban</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>ii. ba</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

D. Learning

(10) The learning problem:

Let \((T, C)\) be a tableau set, and assume that each tableau \( T = (A_{in}, C) \in T \) is finite and contains exactly one designated intended winning candidate \( o \in A_{in} \).

Let \( O \) be the set of all such intended winners. Is there a weighting of the constraints in \( C \) that defines all and only the forms in \( O \) as optimal? If so, what is an example of such a weighting?

(11) 

<table>
<thead>
<tr>
<th>weight</th>
<th>?</th>
<th>?</th>
<th>?</th>
<th>( \mathcal{H} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>( C_1 )</td>
<td>( C_2 )</td>
<td>( C_3 )</td>
<td></td>
</tr>
<tr>
<td>a. Winner</td>
<td>-4</td>
<td>0</td>
<td>-4</td>
<td>?</td>
</tr>
<tr>
<td>b. Loser</td>
<td>0</td>
<td>-2</td>
<td>0</td>
<td>?</td>
</tr>
</tbody>
</table>

(12) \( \mathcal{H}(W) > \mathcal{H}(L) \)

\( \mathcal{H}(W) = (-4 \cdot W(C_1)) + (-4 \cdot W(C_3)) \)

\( \mathcal{H}(L) = -2 \cdot W(C_2) \)
(13) One solution (from the infinite set of possibilities):
\[ W(C_1) = 1, W(C_2) = 4.1, W(C_3) = 1 \]
\[ H(W) = -8 \]
\[ H(L) = -8.2 \]

(14) An inconsistent tableau set:

<table>
<thead>
<tr>
<th>Input 1</th>
<th>C₁</th>
<th>C₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Winner₁</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>b. Loser₁</td>
<td>-6</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input 2</th>
<th>C₁</th>
<th>C₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Winner₂</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>b. Loser₂</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

(15) Linear inequalities with a different notation and some algebra:
\[
(0 \cdot W(C_1)) + (-2 \cdot W(C_2)) > (-6 \cdot W(C_1)) + (0 \cdot W(C_2)) \approx
\[
0w_1 - 2w_2 > -6w_1 + 0w_2 \quad \Rightarrow
\[
-2w_2 > -6w_1 \quad \Rightarrow
\[
-2w_2 + 6w_1 > 0 \quad \Rightarrow
\[
6w_1 - 2w_2 > 0
\]
\[
(-1 \cdot W(C_1)) + (0 \cdot W(C_2)) > (0 \cdot W(C_1)) + (-1 \cdot W(C_2)) \approx
\[
-1w_1 + 0w_2 > 0w_1 - 1w_2 \quad \Rightarrow
\[
-1w_1 > -1w_2 \quad \Rightarrow
\[
-1w_1 + 1w_2 > 0 \quad \Rightarrow
\[
1w_2 - 1w_1 > 0
\]

(16) Feasibility

minimize \(1w_1 + 1w_2\)

Hence: the smallest weight must be greater than 0.

Hence: \[6w_1 - 2w_2 \geq 1\]
\[-1w_1 + 1w_2 \geq 1\]
\[1w_1 \geq 1\]
\[1w_2 \geq 1\]
(17) Solve for the above:

\[
\begin{array}{cccccccc}
& -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
w_1 & -2 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
w_2 & & & & & & & & \\
\end{array}
\]

(18) This is all implemented in R so you can play with the system: `hg.R`. (It’s obviously also implemented in OT-Help.)

E. Predictions

(19) HG allows some tradeoffs.

<table>
<thead>
<tr>
<th>/x/</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>z</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

\[\text{vs.}\]

<table>
<thead>
<tr>
<th>/a/</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

(20) \(-1B > -1A = 1A - 1B > 0 \Rightarrow 1A - 1B \geq 1\)

(21) \(-1A > -2B = -1A + 2B > 0 \Rightarrow -1A + 2B \geq 1\)
(22) Minimize: $1A + 1B$ with constraints:

\[
1A - 1B \geq 1 \\
-A1A + 2B \geq 1 \\
1A \geq 1 \\
1B \geq 1
\]

F. Lango

(23) $[+ATR]$ spreads in both directions:

\[
\begin{align*}
/wot+\epsilon/ & \quad [wode] \quad \text{‘son (3 SG)’} \\
/i\etau+\epsilon/ & \quad [i\etaute] \quad \text{‘neck (3 SG)’} \\
/wot+\alpha/ & \quad [woda] \quad \text{‘son (1 SG)’} \\
/buk+\alpha/ & \quad [bukko] \quad \text{‘book (1 SG)’} \\
/atm+ni/ & \quad [atinni] \quad \text{‘child (2 SG)’} \\
/dek+ni/ & \quad [dekki] \quad \text{‘stew (2 SG)’} \\
/lot+wu/ & \quad [lutwu] \quad \text{‘stick (2 PL)’} \\
/le+wu/ & \quad [lewu] \quad \text{‘axe (2 PL)’}
\end{align*}
\]

(24) High vowel trigger

RL spreading only when the trigger is high:

\[
\begin{align*}
/\eta\epsilon+n+Co/ & \quad [\eta\epsilonnno] \quad *[\eta\epsilonnno] \quad \text{‘to see’} \\
/LR spreading across a cluster only when the trigger is high: & \\
/gwok+\alpha/ & \quad [gwokka] \quad *[gwokka] \quad \text{‘dog (1 SG)’}
\end{align*}
\]

(25) LR directionality:

Mid vowel triggers spread only LR

\[
\begin{align*}
/lm+Co/ & \quad [limmo] \quad *[limmo] \quad \text{‘to visit’} \\
/Spreading from a back trigger across a cluster to a non-high target only LR & \\
/gwok+\alpha/ & \quad [gwokka] \quad *[gwokka] \quad \text{‘dog (1 SG)’}
\end{align*}
\]

(26) Intervening singleton

LR spreading from mid vowels occurs only across a singleton

\[
\begin{align*}
/gwok+\alpha/ & \quad [gwokka] \quad *[gwokka] \quad \text{‘dog (1 SG)’} \\
/RL spreading from a back trigger to a non-high target only across a singleton & \\
/dek+\alpha/ & \quad [dekku] \quad *[dekku] \quad \text{‘stew (2 PL)’}
\end{align*}
\]

(27) High target

RL spreading from a back trigger across a cluster only to high vowels

\[
\begin{align*}
/dek+\alpha/ & \quad [dekku] \quad *[dekku] \quad \text{‘stew (2 PL)’}
\end{align*}
\]
(28) Front trigger
RL spreading across a cluster to a mid target only from a front trigger
/dek+wu/ [dekwu] *[dekwu] ‘stew (2 PL)’

(29) Sprd[ATR]
For any prosodic domain \( x \) containing a vowel specified as ATR, assign a violation mark to each vowel in \( x \) that is not linked to an ATR feature.

(30) Hd-L
Assign a violation mark to every head that is not leftmost in its domain.

(31) Hd[high]
Assign a violation mark to every head that is not high.

(32) Hd[front]
Assign a violation mark to every head that is not front.

(33) Lcl-C
Assign a violation mark to every cluster intervening between a head and a dependent.

(34) ATR[high]
Assign a violation mark to every ATR vowel that is not high.

(35)

<table>
<thead>
<tr>
<th>input</th>
<th>W ∼ L</th>
<th>Sprd[ATR]</th>
<th>11</th>
<th>8</th>
<th>4</th>
<th>Lcl-C</th>
<th>4</th>
<th>2</th>
<th>2</th>
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<tbody>
<tr>
<td>iCe</td>
<td>iCe</td>
<td>iCe ~ iCe</td>
<td>W</td>
<td>L</td>
<td>L</td>
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<td></td>
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<tr>
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<td>uCe</td>
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<td>L</td>
<td>L</td>
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<td></td>
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<td>eCe</td>
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<td>iCe</td>
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<td>W</td>
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<td></td>
<td></td>
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<tr>
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<td>uCCe</td>
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<td>W</td>
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<tr>
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<td>L</td>
<td>W</td>
<td>W</td>
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References