Some Consequences of Natural Law in Syntactic Structure*

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In this paper, we note that a number of unexplained and stipulative properties of the grammar (such as the Generalized Extended Projection Principle, Binary Branching, Labeling) find a functional explanation, if we view them as correlates of a general desire for the grammar to maximize trees in such a way that they result in a Fibonacci-like sequence of maximal categories. Further, we note that the maximal number of arguments available in a clause also corresponds to the Fibonacci series in terms of counting the number of terms involved at each stage of the MERGE derivation.

0. Introduction

Since Chomsky’s (1981) addition of the requirement that every sentence have a subject to the Projection Principle, the apparent stipulative nature of this constraint has been a source of worry for generative grammarians. The empirical basis of the Extended Projection Principle (EPP) is well-established but the external motivations for why Universal Grammar should contain such a requirement are fairly mysterious. This lack of explanation has been compounded in more recent versions of the Minimalist Program, where the EPP has been generalized to all types of specifier positions (e.g., Chomsky 2001).

At the same time, important work in the philosophical foundations of Minimalism has suggested that universal syntactic principles, in particular those governing the simple, mathematical computational

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system, should follow from general physical principles that govern the way biological systems emerge in the phenotype. For example, Uriagereka (1997) has claimed that linguistic structures exhibit the mathematical properties of the “Golden Mean” as exhibited, for example, in the Fibonacci series (0, 1, 1, 2, 3, 5, 8, 13, etc). The Fibonacci series is one of the most interesting mathematical curiosities that pervade in the natural world. For centuries now, observers of nature have recognized that, to take a concrete example, the majority of plants have a number of petals taken from the series. The series matches the one invented around 1200 by Leonardo Fibonacci, who happened to be focusing on a problem about the growth of a population in rabbits. In the Fibonacci series, each number is the sum of the two that precede it. Early approaches to the arrangements of leaves, petals, florets, scales, and the like were purely descriptive. They did not explain how the numbers found relate to the dynamics of growth, they just sorted out the geometry of the arrangements. Recently, mathematical physicists Douady and Couder devised a theory of plant growth and used computer models and laboratory experiments to show that it accounts for the Fibonacci series (see Douady and Couder 1992). Building on work by Vogel (himself taking his lead from work by crystallographers A. and L. Bravais), who had suggested that the observed arrangements followed from efficient packing (space filling), Douady and Couder showed that the series can be seen as a consequence of simple dynamics. The gist of their idea is this: They assumed that successive elements of some kind form at equally spaced intervals of time somewhere on the edge of a small circle, representing the apex, and that these elements then migrate radially at some specified initial velocity. They further assumed that the elements repel each other (similar to electric charges). The repulsion ensures that the radial motion continues and that each new element appears as far as possible from its immediate successors. (Such a system can indeed be shown to satisfy conditions on efficient packing, but crucially that follows from dynamics, not from some postulated constraints on efficiency.)

In this speculative squib, we consider two cases where we see this kind of pattern as an externalist explanation for a number of otherwise unexplained stipulative properties of the grammar. First, we

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1For comprehensive overview of the application of the Fibonacci series in the natural world, see Ball (1997), among many others.
observe that one particular Fibonacci-like sequence in tree structures which are maximized in terms of specifiers and complements might explain why languages aim towards filled specifiers as stipulated by the generalized EPP. This, in turn, we claim makes some interesting predictions about the binarity of the MERGE operation, the ambiguity of terminal complements and the nature of the labeling operation subsumed in MERGE. Second, we contend that in counting the number of heads, complements, specifiers, labels etc. as a derivational sequence, we can explain why the number of arguments found in natural language is limited the way it is; and why the vP is a natural "Phase" in the sense of Chomsky (1999).

1. **A Minor Mathematical Observation.**

Consider a tree, generated by MERGE from the numeration \{A, B, C, D, E, F, G, H, I, J, K, L\} where every specifier and every complement position has been filled\(^2\), and each item has been merged only once. Further, let’s annotate this tree with the maximal categories as defined by Chomsky (1994)—where a label that is immediately dominated by the projection of another category is an XP\(^3\)—and for ease of exposition we annotate other non-terminal nodes as X'. One possible version of such a tree is given in (1). Nodes that are ambiguous between terminal and maximal categories are annotated XP/X.

\[\text{(1)}\]

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AP
 /   \
BP A'
 /   \   /
CP B' A   DP
 /   /   \
EP C' B   FP G P
 /   /   /   \
HP/H E C IP/I JP/J F KP/K G D LP/L
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\(^2\) At the bottom-most layer of such a tree, some nodes may be ambiguous between specifier and complement status. For example, HP/H in (1) is simultaneously a specifier and a complement, in the traditional sense of X-bar theory (cf. Chomsky 1995's slightly different definitions); further branching below this level would disambiguate these structures.

\(^3\) See also the principles of determining maximal and minimal status discussed in Speas (1992)
If we count the number of XPs in each line of this derivation, counting XP/X° ambiguous cases as XPs, we see a partial Fibonacci sequence (1, 1, 2, 3, 5). This property is true of all trees that are maximized in this way. The first (top) level -- the root node -- consists of one XP node. The next level contains one XP (the specifier of the root) and one X’ . Thereafter, in the nth level there is one XP for each YP in the (n–1)th level (the specifier of that YP), and one XP for each YP in level n-2 (complement of that YP); no other XPs occur in the nth level. Letting the function XP(n) represent the number of XPs in the nth level, we see that XP(1) = 1, XP(2) = 1, and for n > 2, XP(n) = XP(n−1) + XP(n−2). Then XP(n) = Fib(n), the Fibonacci function which yields the familiar sequence 1, 1, 2, 3, 5, 8...

Needless to say, real syntactic structures are not maximized in the way (1) is. However, in the ideal case it is clear that ‘maximizing’ the tree entails filling all specifiers. Thus the tendency encoded in the generalized EPP can be derived from this pattern. The fact that real sentences don’t fully maximize in this manner can be seen to be due to the externally imposed limitations of creating a meaningful sentence. As such the EPP can be viewed as a means towards maximizing the tree towards “Fibonacci-hood”, but other factors interfere to limit the degree to which this is achieved. In the world of biology, similar effects can be seen that interrupt idealized Fibonacci qualities. For example, the fractal quality of zebra stripes can be interrupted by the appearance of an ear, or a scar. Likewise, the number of petals in a flower may not fall exactly into the Fibonacci sequence; it is a tendency, not an absolute law. With this in mind, the fact that the sentences of human languages aren’t fully fractal isn’t surprising.

2. Some Further Consequences.

In addition to predicting that specifiers should be filled, a necessary consequence of tree maximization is that XP complements should also apparently be obligatory. This effect may appear to be partly counterintuitive; for example, we are left with the problem that every head must have an XP complement,

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4 The Fibonacci sequence is often claimed to "starts with" a 0, we assume that for reasons of conceptual necessity, there is no “level 0” in a sentence that has some set of nodes, thus the number 0 is absent from the sequence based on a syntactic tree. In principle, a Fibonacci sequence can start anywhere in the set of whole numbers.
but this would mean that we would never have a line in the tree with only terminals. This is clearly a problem since, at least on the face of things, the final line should consist of nothing but terminals.

Nevertheless, we believe that this pressure for the tree to be as Fibonacci-like as possible -- via maximization -- may provide an explanation for a few of the more stipulative properties of the operation MERGE. Let us distinguish three parts of the operation:

\( \text{(2) } \)

a) Take two (and only two) terms  

b) Combine them in a set  

c) Give the set a label.

Of these three parts, only (2b) has a clear motivation (presumably compositionality). Parts (2a) and (2c) appear to be stipulations (However, see Kayne (1984) for a discussion of the motivations of binary branching). We believe that both these properties fall out directly from the functional pressure of tree maximization.

Let us start first with the binarity stipulation in (2a). If MERGE were allowed to optionally select three terms and merge them into a ternary structure, then the fractal Fibonacci sequence of maximal categories would disappear\(^5\). For example, in line 3 of (1) above, if A were to be merged with both DP and another maximal category (say YP) then the sequence would be 1, 1, 3, ... Due to the optionality of two vs. three term merging, it would be impossible to predict where such breaks in the sequence would occur. Similarly, if MERGE was allowed to apply to singletons, then we would predict non-branching structures of the form in (3):

\(^5\) Both Richard Kayne and Terry Langendoen have pointed out to us that this isn't exactly true. While a ternary branching system wouldn't result in the Fibonacci series per se, it would result in a Fibonacci like series. Note, however, that such a structure would require that every head have two sisters, which might be ruled out via the principle of compositionality. On the surface, it appears that such constructions are relatively rare in Grammar, perhaps being limited to double object constructions and applicative-like constructions. Deeper investigation of these kinds of structures have shown that they are, despite surface appearances, clearly underlingly binary; see section 3 below for more on this question.
Chomsky (1994) argues that such structures should be disallowed (see also Carnie 2001, and cf. Kayne 1994).

Interestingly, the conflicting pressures of saying a string of linguistic atoms (words), and the pressure of having a Fibonacci-like sequence of maximal categories results in structures like that in line 5 of (1), where we have elements that are ambiguously XPs and X°s (see Speas 1992 and Carnie 1995, 2001 for empirical arguments that ambiguous structures like these are motivated). It appears, then, that the functional pressure to achieve a fractal structure by tree maximization might explain why non-branching and ternary-plus branching would be disallowed from the MERGE operation.

The fact that constituent structures have labels also appears to be a stipulation. Indeed Collins (2002) claims that this part of the MERGE operation should be abandoned. A discussion of his arguments would take us too far afield. However, we note that, in fact, labels provide the necessary mechanism for determining whether a node is an XP or not (if a node is immediately dominated by a node bearing a different label, then it is an XP). As such if grammar is aiming towards a structure where a Fibonacci sequence of XPs is required, then a mechanism for determining “XP” hood is necessary (contra both Collins 2002 and Carnie 2001). Tree maximization thus provides an external motivation for this part of the MERGE operation as well.

3. **Thematic Structure & Generalized Transformations.**

Recent research on the nature of theta-roles (Hale and Keyser 1993, Borer 1994, Chomsky 1995, Kratzer 1996, Marantz 1993, Pylkkänen 2000, among others) has uncovered a complex structure for the thematic layer, with (in its most articulated and principled form) consists of the following:

\[
\left[\text{vP}\left[\text{Appl-EP}\left[\text{Appl}^E\left[\text{VP}\left[\text{Appl-IP}\left[\text{Appl}^I\left[\text{XP}\right]\right]\right]\right]\right]\right]\right]
\]

Following Marantz and Pylkkänen, we assume that "double object" structures split into two types: a low
individual Applicative type (ApplI), which expresses a relation among individuals, and a high event applicative type, which expresses an individual-event (VP) relation (ApplE). The final layer (vP) introduces the outermost ('Agent') argument (see the work by Kratzer, Chomsky, and others). The point is that (4) has been defined empirically as the maximal thematic domain (forming a 'phase' in Chomsky's 2000 sense). Importantly, there does not seem to be any intrinsic reason semantically or morpho-phonologically as to why (4) should be a maximal space. Surely one could conceive of a system with say 7, 19, or 23 arguments. Such systems are not found in natural languages.

We claim that (4) is also the maximal space defined according to Fibonacci series, now understood in terms of dynamics. To show this, let us proceed stepwise, and work our way through the various concatenation operations that yield (4). The generalized transformation Merge takes two elements X and Y, and forms a pair (X,Y). The steps correspond to the sequence 1 ({X}), 1 ({Y}), 2 ({X,Y}). Assuming binary branching (Kayne 1984), the next operation will add a newly introduced element ({Z}) to the pair (X,Y), resulting in the following configuration.

(6) \[ XP Z \{X Y\} \]

There are two ways of reading (6). Either we decide, with Collins 1999, to eliminate labels (the XP and X on the brackets in (4)), in which case we are left with 3 elements (still a Fibonacci series: 1, 1, 2, 3), or else we take labels into considerations, in which case, we obtain 5 elements, which is still on the Fibonacci series. So far, the minimal, binary-branching structure appears to yield the beginning of a Fibonacci series: 1, 1, 2, 3, 5. Going back to (4), a quick count shows that taking heads, specifiers, and complements together yields a maximal space of 13, which is another number on the Fibonacci series. In fact, as suggested above, there is a sense in which (4) is the maximal space defined by Fibonacci series. As can be immediately gathered, adding a projection to (4) would not allow us to reach the next number in the series (21). Natural laws seem to dictate the end of a series at the vP level, where Chomsky posits a

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Note that intermediate bar-level categories in the sense of Chomsky (1970) are 'left out of the series,' in that \{X\}' is not a 4th distinct element (4 is not on the series), but comes about by the very geometry of the syntactic tree when a specifier, a head, and a complement combine in a way consistent with the most basic (binary) branching pattern. This result concerning intermediate bar-levels is then congenial to the idea expressed in Chomsky (1995) that intermediate bar-levels are invisible for computational purposes.
To the best of our knowledge, no one has a good account of why argument structure should be limited the way it is in natural languages.

4. Speculations and Worries.

In this short paper, we have attempted to offer some speculations about why such constraints and operations as the EPP, labeling, and binary branching might exist in a linguistic system. We have claimed that these things might follow if, as Uriagereka has claimed, the physical properties of the universe push biological systems into particular mathematical patterns. Syntactic structure, like the stripes of zebras and the petals on a flower, strives towards the particular mathematical symmetry found in the Fibonacci sequence. Of course, we have no way of proving these claims. We merely note that the structural requirements needed to produce the Fibonacci sequence (in the ideal case) are precisely some of the most surprising properties of the computational system.

There are, of course, many possible objections to the characterization we have given. One that strikes us as particularly important is the claim that the EPP is not universal. McCloskey (1996) for example, has claimed that some VSO languages appear to lack EPP effects (there is no obligatory movement to subject position, there are no expletives, etc.). If McCloskey’s assertions prove to be correct, then obviously our speculations rest on far less stable ground. However, if alternative explanations for McCloskey’s facts prove tenable, then perhaps our observations offer some insight into the nature of syntactic structure.

In our view, the fact that the observed results match a mathematical series opens up an interesting avenue for future research that may lead to a possible deep explanation. If pressed, I would even say that the possibility of applying the Fibonacci series to linguistic tree geometry suggests that the same growth dynamics found in nature is genuinely at work in language, and that recent views on dynamic linguistic
computation (Chomsky in press, Epstein et al. 1998, Uriagereka 1998, and many others) thereby receive independent support.

**References**


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Interestingly, the next projection up in Chomsky’s (2000) core functional architecture (TP) includes in its domain $T^0$, the specifier of TP, and the so-called edge of the $vP$ phase: 3 elements. The assumption that $vP$ is a phase allows us to get back onto the Fibonacci series. The next level of projection (CP), will add a head $C^0$ and a specifier to the previously gathered set of 3 $\{\text{SpecTP, T, and VP-edge}\}$. If labels are taken into consideration, we obtain the following result:

(7) $[CP \ [C \ C \ [TP \ [T \ T \ VP\text{-edge}]/viewspace]]$

Counting all the elements in (7) yields 9, which is not on the series. However, linguistically speaking, (5) is never found as such. If CP is involved, as in interrogative contexts, one typically finds a movement of T-to-C (subject-auxiliary inversion: $what \ will, \ John \ t, \ do?$). This operation arguably reduces the number of elements by 1 (collapsing two heads), which allows us to get back onto the series by obtaining the number 8. Strikingly, the addition of a projection on top of CP would definitely disrupt the series (the reader can verify for him/herself that an extra head and specifier would never allow us to reach the next

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$^7$Subject questions (e.g., $Who \ came?$) do not involve T-to-C movement, but they are arguably bare IPs (Chomsky 1986), and as such remain part of the series.
number up, 13). But here too, just as in the case of vP, where we had reached a point of saturation, there is good evidence that CP is a phase (Chomsky 2000), in our terms a maximal space. This is another instance where an argument based on linguistic realities converges on an independently identified mathematical pattern.